



## THERMODYNAMIC PROPERTIES OF ROCK-FORMING OXIDES, $\alpha\text{-Al}_2\text{O}_3$ , $\text{Cr}_2\text{O}_3$ , $\alpha\text{-Fe}_2\text{O}_3$ , AND $\text{Fe}_3\text{O}_4$ AT HIGH TEMPERATURES AND PRESSURES

P. I. Dorogokupets<sup>1</sup>, T. S. Sokolova<sup>1</sup>, A. M. Dymshits<sup>2</sup>, K. D. Litasov<sup>2,3</sup>

<sup>1</sup> Institute of the Earth's Crust, Siberian Branch of RAS, Irkutsk, Russia

<sup>2</sup> V.S. Sobolev Institute of Geology and Mineralogy, Siberian Branch of RAS, Novosibirsk, Russia

<sup>3</sup> Novosibirsk State University, Novosibirsk, Russia

**Abstract:** Equations of state of corundum ( $\alpha\text{-Al}_2\text{O}_3$ ), eskolaite ( $\text{Cr}_2\text{O}_3$ ), hematite ( $\alpha\text{-Fe}_2\text{O}_3$ ), and magnetite ( $\text{Fe}_3\text{O}_4$ ) are constructed based on the Helmholtz free energy by simultaneous optimization of ultrasonic, X-ray diffraction, dilatometric, and thermochemical measurements. The magnetic contribution to  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$  Helmholtz free energy was determined via the A.T. Dinsdale model [Dinsdale, 1991]. The calculated thermodynamic properties of rock-forming oxides of aluminum, chromium, and iron are in good agreement with the reference data and experimental measurements at room pressure, as well as with  $P\text{-}V\text{-}T$  measurements at high temperatures and pressures. Thermodynamic functions ( $x$ ,  $\alpha$ ,  $S$ ,  $C_P$ ,  $C_V$ ,  $K_T$ ,  $K_S$ ,  $\gamma_{\text{th}}$ ,  $G$ ) of corundum, eskolaite, hematite, and magnetite are calculated at different pressures (up to 80, 70, 50 and 20 GPa, respectively) and temperatures (up to 2000 K), and the results are tabulated. The calculated Gibbs energy of rock-forming oxides can be used to construct the phase diagrams of mineral systems, which include the oxides under the conditions of the Earth's mantle.

**Key words:** thermodynamics; equation of state; Helmholtz free energy; oxide; corundum; eskolaite; hematite; magnetite; mantle

**Recommended by** E.V. Sklyarov

**For citation:** Dorogokupets P.I., Sokolova T.S., Dymshits A.M., Litasov K.D. 2016. Thermodynamic properties of rock-forming oxides,  $\alpha\text{-Al}_2\text{O}_3$ ,  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$  at high temperatures and pressures. *Geodynamics & Tectonophysics* 7 (3), 459–476. doi:10.5800/GT-2016-7-3-0217.

## ТЕРМОДИНАМИЧЕСКИЕ СВОЙСТВА ПОРОДООБРАЗУЮЩИХ ОКСИДОВ $\alpha\text{-Al}_2\text{O}_3$ , $\text{Cr}_2\text{O}_3$ , $\alpha\text{-Fe}_2\text{O}_3$ И $\text{Fe}_3\text{O}_4$ ПРИ УСЛОВИЯХ ВЫСОКИХ ТЕМПЕРАТУР И ДАВЛЕНИЙ

П. И. Дорогокупец<sup>1</sup>, Т. С. Соколова<sup>1</sup>, А. М. Дымшиц<sup>2</sup>, К. Д. Литасов<sup>2,3</sup>

<sup>1</sup> Институт земной коры СО РАН, Иркутск, Россия

<sup>2</sup> Институт геологии и минералогии им. В.С. Соболева СО РАН, Новосибирск, Россия

<sup>3</sup> Новосибирский государственный университет, Новосибирск, Россия

**Аннотация:** На основе свободной энергии Гельмгольца построены уравнения состояния корунда ( $\alpha\text{-Al}_2\text{O}_3$ ), эсcolaита ( $\text{Cr}_2\text{O}_3$ ), гематита ( $\alpha\text{-Fe}_2\text{O}_3$ ) и магнетита ( $\text{Fe}_3\text{O}_4$ ) путем одновременной оптимизации ультразвуковых, рентгеновских, дилатометрических данных и термохимических измерений теплоемкости при атмосферном давлении и при повышенных температурах и давлениях. Магнитный вклад в свободную энергию Гельмгольца для  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$  и  $\text{Fe}_3\text{O}_4$  определен с помощью модели А.Т. Динсдэла [Dinsdale, 1991]. Предложенный подход к построению уравнений состояния хорошо описывает  $\lambda$ -видную аномалию в теплоемкостях эс-

колаита, гематита и магнетита, которая связана с изменением магнитных свойств. Полная термодинамическая модель уравнений состояния  $\alpha\text{-Al}_2\text{O}_3$ ,  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$  и  $\text{Fe}_3\text{O}_4$  содержит группу из семи фиксированных параметров и группу из девяти подгоночных параметров, значения которых определяются методом наименьших квадратов. Рассчитанные термодинамические функции породообразующих оксидов алюминия, хрома и железа хорошо согласуются со справочными данными и экспериментальными измерениями при атмосферном давлении, а также с современными  $P\text{-}V\text{-}T$  измерениями в алмазных наковальнях и многопулансонных аппаратах высокого давления. Приведена табуляция термодинамических функций (объем, коэффициент термического расширения, изобарная и изохорная теплоемкость, энтропия, адиабатический и изотермический модули сжатия, термодинамический параметр Грюнейзена и энергия Гиббса) корунда, эсколаита, гематита и магнетита до температуры 2000 К при разных давлениях (до 80, 70, 50 и 20 ГПа, соответственно). Таким образом, полученные уравнения состояния уточняют термодинамику оксидных фаз от стандартных условий до температур и давлений, соответствующих условиям мантии Земли. Рассчитанная энергия Гиббса породообразующих оксидов алюминия, хрома и железа может быть использована для построения фазовых диаграмм минеральных систем с их участием, имеющих принципиальное значение для интерпретации глобальных и промежуточных границ в земной мантии.

**Ключевые слова:** термодинамика; уравнение состояния; свободная энергия Гельмгольца; оксид; корунд; эсколаит; гематит; магнетит; мантия

## 1. INTRODUCTION

Transition-metal sesquioxides have been already studied over a wide range of temperatures and pressures as their electrical and magnetic properties are widely variable. Corundum ( $\alpha\text{-Al}_2\text{O}_3$ ), eskolaite ( $\text{Cr}_2\text{O}_3$ ), hematite ( $\alpha\text{-Fe}_2\text{O}_3$ ) and iron-bearing compounds with related structures ( $\text{Fe}_3\text{O}_4$ ) also play important role in the geology of the Earth interior, and ruby,  $(\text{Al},\text{Cr})_2\text{O}_3$  was used as a pressure calibration scale for *in situ* diamond anvil cell (DAC) studies. Considering these applications, phase and magnetic transformations, as well as thermodynamic properties of corundum type oxides are of great interest both for fundamental and applied sciences.

Many experiments were conducted to study the high-pressure behavior of corundum-type compounds. It was shown that  $\alpha\text{-Al}_2\text{O}_3$  transforms to the  $\text{Rh}_2\text{O}_3(\text{II})$ -type structure (space group  $Pbcn$ ) at  $\sim 80$  GPa [Lin *et al.*, 2004] and to the  $\text{CaIrO}_3$ -type phase ("post-perovskite", space group  $Cmcm$ ) above 130 GPa [Oganov, Ono, 2005; Ito *et al.*, 2009]. A further phase transition to a  $\text{U}_2\text{S}_3$ -type polymorph (space group  $Pnma$ ) at  $\sim 370$  GPa [Umemoto, Wentzcovitch, 2008] and new thermodynamically stable compounds in the system Al-O above 330 GPa [Liu *et al.*, 2015] are predicted. To date, five different crystalline polymorphs of  $\text{Fe}_2\text{O}_3$  have been discovered and described [Tucek *et al.*, 2015]. At ambient conditions,  $\alpha\text{-Fe}_2\text{O}_3$  crystallizes in the rhombohedral corundum-type structure (space group  $R\bar{3}c$ ). At room temperature with increasing pressure, above  $\sim 50$  GPa,  $\text{Fe}_2\text{O}_3$  forms a novel monoclinic phase with space group  $P2_1/n$  and, above 67 GPa, compression triggers the transition to a different HP phase with the orthorhombic unit cell and space group  $Aba2$ .

The pressure-induced  $\text{Fe}^{3+}$  high-spin to low-spin transition was monitored accompanying the change in the crystal structure [Ono, Ohishi, 2005]. At ambient pressure, both  $\alpha\text{-Fe}_2\text{O}_3$  and  $\text{Cr}_2\text{O}_3$  are insulators and antiferromagnetic below the Neel temperature ( $T_N$ ) of 960 and 311.5 K, respectively [Gronvold, Samuels, 1975; Worlton *et al.*, 1968]. The electronic and magnetic properties of  $\text{Cr}_2\text{O}_3$  under pressure were studied using a diamond anvil cell at pressures up to 55 GPa, and the evidence for two discontinuous transitions of electronic or magnetic nature, most likely associated with the change in magnetic ordering and charge transfer, were reported at  $\sim 15$  to 30 GPa [Dera *et al.*, 2011]. In the room-temperature compression experiment, the  $\text{Cr}_2\text{O}_3$  remains the original rhombohedral structure up to 70 GPa [Kantor *et al.*, 2012]. An orthorhombic phase was detected after heating at 30 GPa [Shim *et al.*, 2004].

Magnetite ( $\text{Fe}_3\text{O}_4$ ) at ambient conditions is a mixed-valence iron oxide and belongs to the spinel group of minerals and crystallizes in a cubic structure (space group  $Fd\bar{3}m$ ). At room temperature and under pressure of  $\sim 21$  GPa,  $\text{Fe}_3\text{O}_4$  transforms into distorted-cubic phase ( $\text{h-Fe}_3\text{O}_4$ ) and this structure has been widely discussed. The recent *in situ* study showed that  $\text{Fe}_3\text{O}_4$  forms a mixture of  $\text{Fe}_4\text{O}_5$  and hematite at 9.5–11 GPa and 973–1673 K, which must recombine to distorted-cubic phase at yet higher pressures [Woodland *et al.*, 2012]. A following transition of  $\text{h-Fe}_3\text{O}_4$  into  $\text{FeO}$  and hematite assemblage and a new structural transition to orthorhombic structure (space group  $Pnma$ ) are debated [Ricolleau, Fei, 2016]. The conducting and magnetic properties of magnetite were studied by different methods (Mössbauer spectroscopic, X-ray diffraction, electrical resistivity measurements), which show that  $\text{Fe}_3\text{O}_4$  and its high-pressure polymorph are ferromag-

netic in the stability field and to at least ~70 GPa, respectively [Shebanova, Lazor, 2003].

The properties of corundum-type compounds have been studied under various  $P$ - $T$  conditions. Calorimetric measurements of enthalpies of formation are reported in [Moore, Kelley, 1944; Gronvold, Westrum, 1959; Westrum, Gronvold, 1969; Gronvold, Sveen, 1974; Gronvold, Samuelsen, 1975; Goto et al., 1989; et al.]. Results of the theoretical studies of elastic properties, crystal structure, thermodynamics and stability of corundum-type oxides are presented in [Sivasubramanian et al., 2001; Wessel, Dronskowski, 2013]. Heat capacities at ambient pressure of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub> [Hemingway, 1990; Klemme et al., 2000; Gurevich et al., 2009; Snow et al., 2010] and thermal expansion [Wachtman, 1962; Schauer, 1965; Skinner, 1966; White, Roberts, 1983; Aldebert, Traverse, 1984; Hill et al., 2010; Dymshits et al., 2016; et al.] were studied experimentally in detail. Elastic properties of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub> were experimentally

studied as a function of  $P$  and  $T$  using X-ray diffraction and ultrasonic technique [Wilburn et al., 1978; Sato, Akimoto, 1979; Finger, Hazen, 1980; Nakagiri et al., 1986; Richet et al., 1998; Dubrovinsky et al., 1998; Kim-Zajonz et al., 1999; Grevel et al., 2000; Reichmann, Jacobsen, 2004; Gatta et al., 2007; Dera et al., 2011; Kantor et al., 2012; Dewaele, Torrent, 2013; Dymshits et al., 2016], and their more detailed description is provided below (see section 3 and 4).

Based on the current experimental results, an internally consistent set of thermochemical and thermophysical data can be consolidated. The thermodynamic analysis can provide internal consistency of different types of physical property measurements and give the complete set of data on a system, for example, based on the Helmholtz free energy. The aim of this paper is to set up the equations of state (EoS) of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub> and calculate the full set of thermodynamic functions depending on temperature and pressure.

## 2. THERMODYNAMIC MODEL OF EoS

The proposed thermodynamic model of equations of state of corundum, eskolaite, hematite, and magnetite is based on a modified formalism from our previous studies [Dorogokupets et al., 2012, 2014, 2015; Sokolova et al., 2013, 2016] and takes into account the magnetic contribution.

The Helmholtz free energy of sesquioxides in classical form [Zharkov, Kalinin, 1971]:

$$F(V, T) = U_0 + E_0(V) + F_{th}(V, T) - F_{th}(V, T_0) + F_{anh}(V, T) - F_{anh}(V, T_0) + F_{mag}(T) - F_{mag}(T_0), \quad (1)$$

where  $U_0$  is the reference energy;  $E_0(V)$  is the potential part of the Helmholtz free energy on reference isotherm  $T_0=298.15$  K, which depends only on volume;  $F_{th}(V, T)$  is the thermal part of the Helmholtz free energy, which depends on volume and temperature;  $F_{anh}(V, T)$  is the contribution of intrinsic anharmonicity to the Helmholtz free energy, which depends on volume and temperature; and  $F_{mag}(T)$  is the magnetic contribution to the Helmholtz free energy, which depends only on temperature.

In physics of metals, a potential part of free energy is often described using the well-known Vinet equation [Vinet et al., 1987], which defines the potential components of EoS  $E_0(V)$ ,  $P_0(V)$ ,  $K_{T0}(V)$ , and  $K_0'$  depending on volume, as:

$$E_0(V) = 9K_0V_0\eta^{-2}(1 - [1 - \eta(1 - y)]\exp[(1 - y)\eta]), \quad (2.1)$$

$$P_0(V) = 3K_0y^{-2}(1 - y)\exp[(1 - y)\eta], \quad (2.2)$$

$$K_{T0}(V) = K_0y^{-2}[1 + (\eta y + 1)(1 - y)]\exp[(1 - y)\eta], \quad (2.3)$$

$$K_0' = \frac{1}{3}\left[2 + y\eta + \frac{y(1-\eta)+2y^2\eta}{1+(1-y)(1+y\eta)}\right], \quad (2.4)$$

where  $y=(V/V_0)=x^{1/3}$ , and  $\eta=1.5(K_0'-1)$ .

As shown earlier [Dorogokupets, Dewaele, 2007; Dorogokupets, 2010], thermodynamic functions at temperatures above the reference isotherm (>298.15 K) can be calculated using the Debye or Einstein model. For a more accurate calculation of standard entropy, we use the Einstein model with two characteristic temperatures. The thermal part of the Helmholtz free energy is expressed as a sum of two Einstein temperature contributions and the contribution of intrinsic anharmonicity:

$$F_{th}(V, T) = m_1 RT \ln \left( 1 - \exp \frac{-\Theta_1}{T} \right) + m_2 RT \ln \left( 1 - \exp \frac{-\Theta_2}{T} \right) + \left( -\frac{3}{2} n R a_0 x^m T^2 \right), \quad (3)$$

where  $\Theta_1$  и  $\Theta_2$  are characteristic temperatures depending only on volume;  $m_1+m_2=3n$ ,  $n$  is number of atoms in a chemical formula of compound;  $a_0$  is an intrinsic anharmonicity parameter;  $m$  is an anharmonic analogue of the Grüneisen parameter;  $R$  the gas constant ( $R=8.31451 \text{ Jmol}^{-1}\text{K}^{-1}$ ).

Hereafter, for simplicity, let us limit ourselves to one characteristic temperature  $\Theta_i$  ( $i=1, 2$ ). Differentiating eq. (3) with respect to temperature at constant volume, we obtain entropy, and calculate internal energy:

$$S = - \left( \frac{\partial F_{th}}{\partial T} \right)_V = 3nR \left[ -\ln \left( 1 - \exp \frac{-\Theta_i}{T} \right) + \frac{\Theta_i/T}{\exp(\Theta_i/T)-1} \right] + 3nRa_0x^mT, \quad (4)$$

$$E_{th} = F_{th} + TS = 3nR \left[ \frac{\Theta_i}{\exp(\Theta_i/T)-1} \right] + \frac{3}{2} n Ra_0 x^m T^2. \quad (5)$$

Differentiating eq. (3) with respect to volume at constant temperature, we determine the thermal part of pressure:

$$P_{th} = - \left( \frac{\partial F_{th}}{\partial V} \right)_T = 3nR \frac{\gamma}{V} \left[ \frac{\Theta_i}{\exp(\Theta_i/T)-1} \right] + \frac{3}{2} n Ra_0 x^m T^2 \frac{m}{V}. \quad (6)$$

Differentiating eq. (5) with respect to temperature at constant volume and eq. (6) with respect to volume at constant temperature, we obtain isochoric heat capacity and isothermal bulk modulus:

$$C_V = \left( \frac{\partial E_{th}}{\partial T} \right)_V = 3nR \left[ \left( \frac{\Theta_i}{T} \right)^2 \frac{\exp(\Theta_i/T)}{[\exp(\Theta_i/T)-1]^2} \right] + 3nRa_0x^mT, \quad (7)$$

$$\begin{aligned} K_{Tth} &= -V \left( \frac{\partial P_{th}}{\partial V} \right)_T = 3nR \left[ \begin{aligned} &\frac{\gamma}{V} (1 + \gamma - q) \left[ \frac{\Theta_i}{\exp(\Theta_i/T)-1} \right] - \gamma^2 \frac{T}{V} \left[ \left( \frac{\Theta_i}{T} \right)^2 \frac{\exp(\Theta_i/T)}{(\exp(\Theta_i/T)-1)^2} \right] + \\ &+ \frac{1}{2} a_0 x^m T^2 \frac{m}{V} (1 - m) \end{aligned} \right] = \\ &= P_{th} (1 + \gamma - q) - \gamma^2 C_V \frac{T}{V} + \frac{3}{2} n Ra_0 x^m T^2 \frac{(mq - my - m^2 + 2\gamma^2)}{V}. \end{aligned} \quad (8)$$

Differentiating eq. (6) with respect to temperature at constant volume, we determine slope:

$$(\partial P_{th}/\partial T)_V = 3nR \frac{\gamma}{V} \left[ \left( \frac{\Theta_i}{T} \right)^2 \frac{\exp(\Theta_i/T)}{(\exp(\Theta_i/T)-1)^2} \right] + 3nRa_0x^mT \frac{m}{V}. \quad (9)$$

The volume dependence of the characteristic temperatures in eq. (3), and unknown parameters  $\gamma$  and  $q$  in eq. (6, 8, 9) are taken from [Al'tshuler et al., 1987]:

$$\Theta_i = \Theta_{0i} x^{-\gamma_\infty} \exp \left[ \frac{\gamma_0 - \gamma_\infty}{\beta} (1 - x^\beta) \right], \quad (10.1)$$

$$\gamma = - \left( \frac{\partial \ln \Theta_i}{\partial \ln V} \right)_T = \gamma_\infty + (\gamma_0 - \gamma_\infty) x^\beta, \quad (10.2)$$

$$q = \left( \frac{\partial \ln \gamma}{\partial \ln V} \right)_T = \beta x^\beta \frac{\gamma_0 - \gamma_\infty}{\gamma}, \quad (10.3)$$

where  $\Theta_{0i}$  is characteristic temperature under standard conditions ( $i = 1, 2$ );  $\gamma_0$  is the Grüneisen parameter under standard conditions;  $\gamma_\infty$  is the Grüneisen parameter at infinite compression, when  $x \rightarrow 0$ ;  $\beta$  is an additional parameter.

To calculate the magnetic contribution to the Helmholtz free energy, we use the formalism from [Dinsdale, 1991], which was modified in [Jacobs, Schmid-Fetzer, 2010] to obtain the correct limit of the entropy at 0 K. The magnetic contribution is expressed as:

$$F_{mag}(T) = RT \ln(B_0 + 1)(g(\tau) - 1), \quad (11)$$

where  $B_0$  is average magnetic moment per atom and  $\tau = T/T_C$ ,  $T_C$  is Curie temperature.

The function  $g(\tau)$  is obtained as:

$$g(\tau) = 1 - \left[ \frac{79\tau^{-1}}{140p} + \frac{474}{497} \left( \frac{1}{p} - 1 \right) \left( \frac{\tau^3}{6} + \frac{\tau^9}{135} + \frac{\tau^{15}}{600} \right) \right] / D, \text{ if } \tau \leq 1, \quad (12.1)$$

$$g(\tau) = - \left( \frac{\tau^{-5}}{10} + \frac{\tau^{-15}}{315} + \frac{\tau^{-25}}{1500} \right) / D, \text{ if } \tau > 1. \quad (12.2)$$

The value  $D$  in eq. (12.1 and 12.2) is calculated as:

$$D = \frac{518}{1125} + \frac{11692}{15975} \left( \frac{1}{p} - 1 \right), \quad (13)$$

where  $p$  is fraction of magnetic enthalpy.

The magnetic contribution in eq. (11) does not depend on pressure, so it will be equal for the Helmholtz free energy and Gibbs energy. The equations for the magnetic contribution to entropy, enthalpy and heat capacity are available in [Dinsdale, 1991]. The same formalism was used to construct the equation of state of bcc-Fe [Dorogokupets et al., 2014]. Eq. (11) is used to determine the magnetic contribution to one atom of chromium or iron. Therefore, the magnetic contribution in EoSes of eskolaite and hematite can be calculated with a multiplier of 2, while the multiplier for magnetite is 3. Parameters  $B_0$  and  $p$  in eq. (11, 13) for  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$  are determined by fitting the heat capacity in the region  $\lambda$ -anomaly.

The total pressure and isothermal bulk modulus are calculated as the sums of potential and thermal parts:  $P=P_0(V)+P_{th}(V,T)$ ,  $K_T=K_{T0}(V)+K_{Tth}(V,T)$ , respectively. Then we can calculate the volume coefficient of thermal expansivity  $\alpha=(\partial P_{th}/\partial T)_V/K_T$ , heat capacity at constant pressure  $C_P=C_V+\alpha^2 TV K_T$ , adiabatic bulk modulus  $K_S=K_T+VT(\alpha K_T)^2/C_V$ , and thermodynamic Grüneisen parameter  $\gamma_{th}=\alpha V K_T/C_V=\alpha V K_S/C_P$ . Enthalpy and the Gibbs energy are determined from the linear relationships:  $H=E+PV$ ,  $G=F(V, T)+PV$ , respectively.

Adding up the corresponding functions, we obtain a complete thermodynamic description of equation of state. Equations (1–13) contain fixed parameters  $U_0$ ,  $V_0$ ,  $K_0$ ,  $K'$ ,  $m_1$ ,  $m_2$ ,  $R$ ,  $T_C$ , and the group of fitted parameters  $\Theta_{01}$ ,  $\Theta_{02}$ ,  $\gamma_0$ ,  $\gamma_\infty$ ,  $\beta$ ,  $a_0$ ,  $m$ ,  $B_0$ , and  $p$ , which are derived by the least squares method. Table 1 shows the EoSes parameters of rock-forming oxides ( $\alpha\text{-Al}_2\text{O}_3$ ,  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$ ), which are obtained by simultaneous optimization of experimental measurements of heat capacity, molar volume, thermal expansion, adiabatic bulk modulus at ambient pressure, and  $P\text{-}V\text{-}T$  measurements at the reference isotherm and at high temperatures.

Table 1. Parameters of equations of state of rock-forming oxides

Таблица 1. Параметры уравнений состояния породообразующих оксидов

| Parameters                            | $\alpha\text{-Al}_2\text{O}_3$ | $\text{Cr}_2\text{O}_3$ | $\alpha\text{-Fe}_2\text{O}_3$ | $\text{Fe}_3\text{O}_4$ |
|---------------------------------------|--------------------------------|-------------------------|--------------------------------|-------------------------|
| $U_0$ , $\text{kJ mol}^{-1}$          | -1690.49                       | -1161.25                | -851.78                        | -1158.10                |
| $V_0$ , $\text{cm}^3 \text{mol}^{-1}$ | 25.575                         | 29.057                  | 30.274                         | 44.580                  |
| $K_0$ , GPa                           | 252.3                          | 211.7                   | 202.5                          | 181.2                   |
| $K'$                                  | 4.14                           | 5.35                    | 3.21                           | 4.90                    |
| $\Theta_{10}$ , K                     | 956                            | 718                     | 554                            | 613                     |
| $m_1$                                 | 7.5                            | 7.5                     | 11.9                           | 10.5                    |
| $\Theta_{20}$ , K                     | 475                            | 376                     | 201                            | 252                     |
| $m_2$                                 | 7.5                            | 7.5                     | 3.1                            | 10.5                    |
| $\gamma_0$                            | 1.323                          | 1.388                   | 2.084                          | 1.341                   |
| $B$                                   | 1.725                          | 0.5                     | 1.3                            | 0.9                     |
| $a_0$ , $10^{-6}\text{K}^{-1}$        | 8.2                            | -                       | -                              | 43.7                    |
| $m$                                   | 1.0                            | -                       | -                              | 1.0                     |
| $T_C$ , K                             | -                              | 307                     | 950                            | 845.5                   |
| $B_0$                                 | -                              | 1.034                   | 1.927                          | 3.751                   |
| $p$                                   | -                              | 0.378                   | 0.304                          | 0.321                   |

### 3. EQUATION OF STATE OF $\alpha$ -Al<sub>2</sub>O<sub>3</sub>

The equation of state of corundum is constructed according to the thermodynamic model using eq. (1–10). Figures 1–3 show the calculated thermodynamic functions of corundum depending on temperature at 1 bar pressure in comparison with various experimental measurements. The calculated heat capacity of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> (Fig. 1) is in good agreement with measurements from [Goto *et al.*, 1989; Archer, 1993] and reference data from [Gurvich *et al.*, 1981; Chase, 1998]. The calculated coefficient of volumetric thermal expansivity (Fig. 2) is in close agreement with measurements from [Wachtman *et al.*, 1962; Schauer, 1965; Kirby *et al.*, 1972; White, Roberts, 1983], but slightly differs from the measurements from [Aldebert, Traverse, 1984; Goto *et al.*, 1989; Saxena, Shen, 1992] at temperatures above 1200 K. The calculated adiabatic and isothermal bulk modulus (Fig. 3) is consistent with the data from [Chung, Simmons, 1968; Goto *et al.*, 1989; Anderson, Isaak, 1995].

The calculated parameters of the reference isotherm of corundum are based on the quasi-hydrostatic measurements in a helium pressure-transmitting medium from [Dewaele, Torrent, 2013]. Pressure is measured using the ruby scale from [Dorogokupets, Oganov, 2007], which slightly underestimates pressure compared to the new calibration of the ruby pressure scale in [Dorogokupets *et al.*, 2012; Sokolova *et al.*, 2013]. The parameters of the equation of state of corundum according [Dewaele, Torrent, 2013] are as follows:  $V_0=25.64 \text{ cm}^3\text{mol}^{-1}$ ,  $K_0=254.1 \text{ GPa}$ , and  $K'=4.00$  (Rydberg-Vinet equation, the ruby scale from [Dorogokupets, Oganov, 2007]). All the experimental  $P$ - $V$ - $T$  measurements are given in the form of  $x=V/V_0$ , where  $V_0$  is the proposed reference value. Experimental data from [Dewaele, Torrent, 2013] are calculated based on the ruby pressure scale from [Sokolova *et al.*, 2013] and using  $V_0=25.575 \text{ cm}^3\text{mol}^{-1}$  from [Robie *et al.*, 1978]. We have obtained for corundum:  $K_0=252.3 \text{ GPa}$ ,  $K'=4.14$ , and these values are in good agreement with the ultrasonic measurements reported in [Chung, Simmons, 1968; Goto *et al.*, 1989; Anderson, Isaak, 1995]. Figure 4 shows that the calculated reference isotherm of corundum is consistent with the recalculated data from [Dewaele, Torrent, 2013], but earlier measurements [Richet *et al.*, 1988; Dubrovinsky *et al.*, 1998; Jephcoat *et al.*, 1988; Funamori, Jeanloz, 1997] give a higher pressure. It should be noted that our calculations are in good agreement with the first studies of corundum compressibility [d'Amur *et al.*, 1978; Finger, Hazen, 1978] in the classic hydrostatic conditions (4:1 volume mixture of methanol-ethanol). Figure 5 shows the deviation of the high-temperature measurements of compressibility [Dubrovinsky *et al.*, 1998; Grevel *et al.*, 2000] from our calculations. In [Grevel *et al.*, 2000], pressure was calculated using NaCl pressure marker from

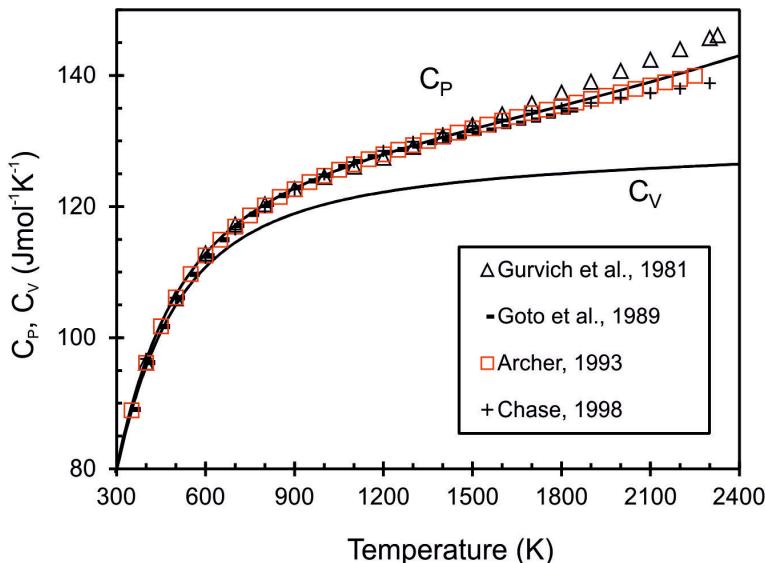
[Decker, 1971]. According to [Strässle *et al.*, 2014], this scale underestimates pressure up to 0.5 GPa (at 10–15 GPa) in comparison with NaCl EoS from [Brown, 1999]. Values more consistent with our EoS calculations for corundum can be obtained by recalculating the data from [Grevel *et al.*, 2000] using NaCl scale from [Brown, 1999]. The data from [Dubrovinsky *et al.*, 1998] significantly overstate the pressure probably due to the pressure measurement method.

The calculated thermodynamic functions of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> depending on temperature at pressures 0.0001, 50, and 80 GPa are presented in Table 2. The last two columns show the increments of the Gibbs energy of corundum at given temperatures and pressures, which are calculated from our data and database in [Holland, Powell, 2011] ( $G$  and  $G^*$ , respectively). The Gibbs energy under standard conditions is calculated with regard to  $\Delta H_{298}=-1675.33 \text{ kJmol}^{-1}$  [Holland, Powell, 2011] and standard entropy  $S_{298}=50.841 \text{ Jmol}^{-1}\text{K}^{-1}$  (Table 2) as follows:  $G_{298}=\Delta H_{298}-S_{298}*298.15$ . In our equation of state, the Gibbs energy under standard conditions is defined by parameter  $U_0$ , therefore  $G_{298}=U_0=-1690.49 \text{ kJmol}^{-1}$ . The calculated  $P$ - $V$ - $T$  relations of corundum can be used for pressure calculations at given temperatures and volumes, which is important for practical aspects of the high-pressure experiments.

### 4. EQUATIONS OF STATE OF Cr<sub>2</sub>O<sub>3</sub>, $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, AND Fe<sub>3</sub>O<sub>4</sub>

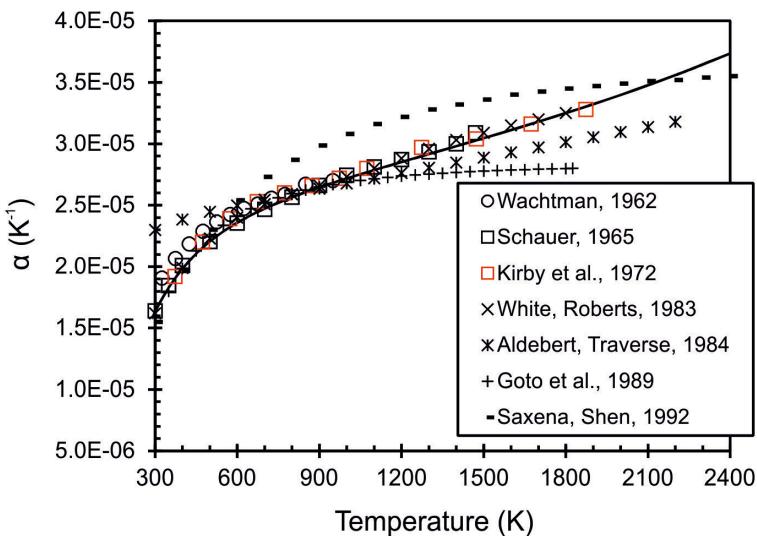
The magnetic contribution to the Helmholtz free energy is considered in the equations of state of esko-laitite, hematite and magnetite using eq. (1–13). A specific  $\lambda$ -type anomaly in the heat capacity of minerals is indicative of the magnetic transition of atoms or rare structural changes as shown for  $\alpha$ -SiO<sub>2</sub> → coesite transformation [Dorogokupets, 1995]. Curie point ( $T_c$ ) determines the critical state that marks a sharp change in the magnetic properties of minerals.

Figures 6–8 show the calculated heat capacities of Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub> in comparison with the experimental measurements. The shape of the  $\lambda$ -type anomaly in the heat capacity of oxides is well described by the EoS suggested in this study. The calculated heat capacity of Cr<sub>2</sub>O<sub>3</sub> is in good agreement with measurements reported in [Bruce, Cannel, 1977; Klemme *et al.*, 2000; Ziemniak *et al.*, 2007; Gurevich *et al.*, 2009; Aristova, Gusarov, 2008]. The heat capacities of  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> and Fe<sub>3</sub>O<sub>4</sub> are consistent with the data from the experiments described in [Gronvold, Westrum, 1959; Westrum, Gronvold, 1969; Gronvold, Sveen, 1974; Gronvold, Samuelsen, 1975; Hemingway, 1990; Shebanova, Lazor, 2003; Snow *et al.*, 2010] and the reference data [Robie *et al.*, 1978]. Cr<sub>2</sub>O<sub>3</sub> undergoes antiferromagnetic-paramagnetic transition at  $T_c=307 \text{ K}$ . The temperature of this magnetic transition is close to 300 K,



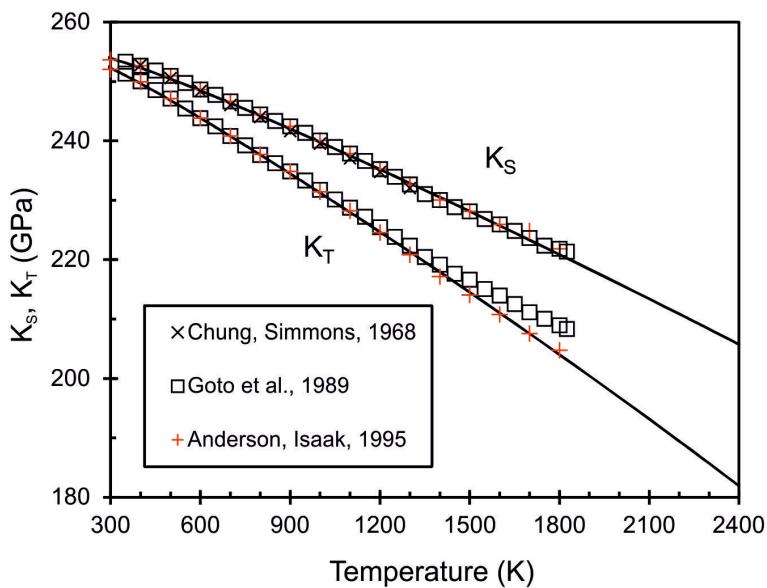
**Fig. 1.** Calculated isobaric ( $C_P$ ) and isochoric ( $C_V$ ) heat capacity of  $\alpha\text{-Al}_2\text{O}_3$  (solid lines) in comparison with selected reference and experimental data [Gurvich et al., 1981; Goto et al., 1989; Archer, 1993; Chase, 1998].

**Рис. 1.** Рассчитанная изобарная ( $C_P$ ) и изохорная ( $C_V$ ) теплоемкость  $\alpha\text{-Al}_2\text{O}_3$  (линии) в сравнении со справочными и экспериментальными данными [Gurvich et al., 1981; Goto et al., 1989; Archer, 1993; Chase, 1998].



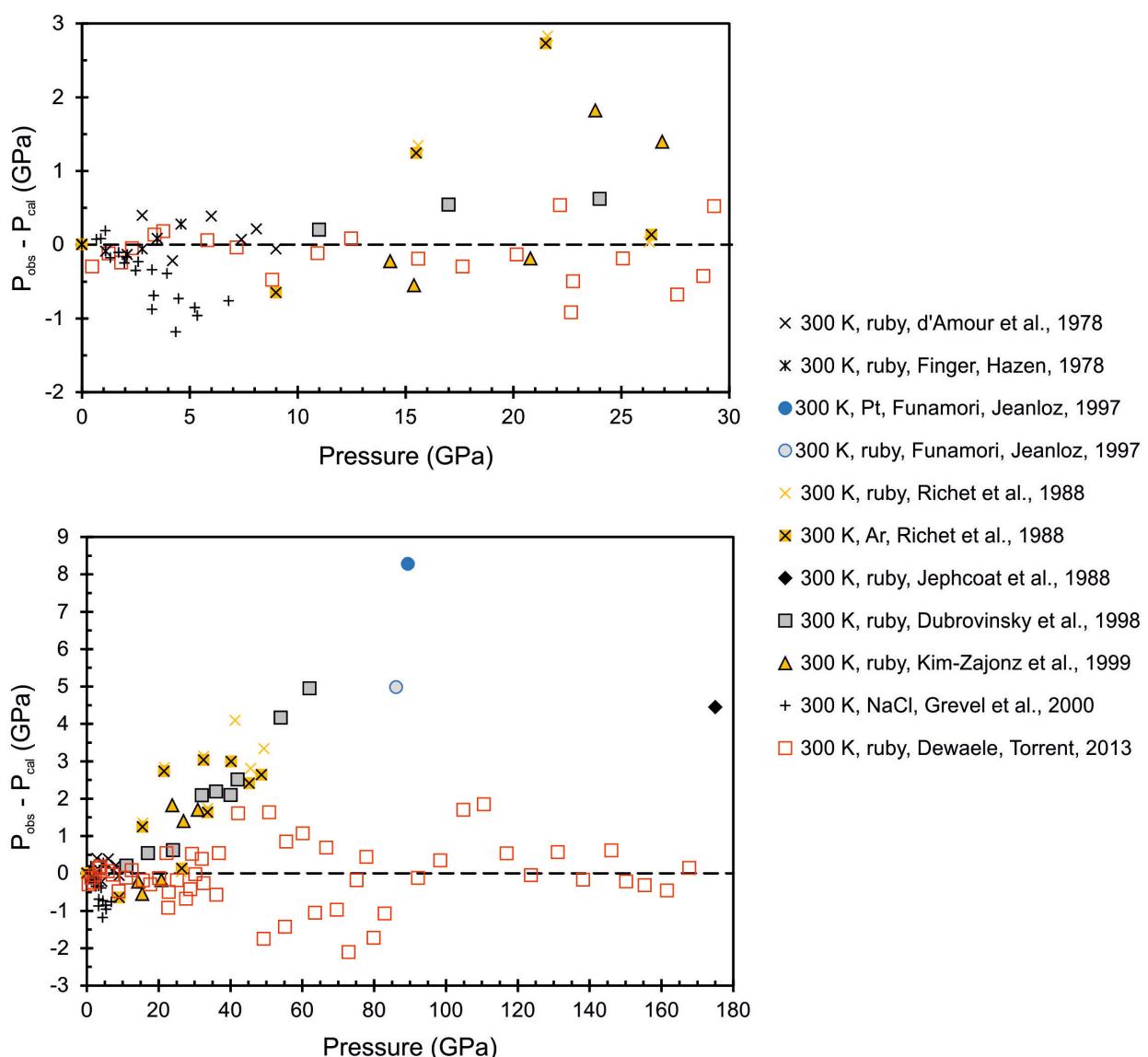
**Fig. 2.** Calculated coefficient of volumetric thermal expansion of  $\alpha\text{-Al}_2\text{O}_3$  (solid line) in comparison with selected reference and experimental data [Wachtman et al., 1962; Schauer, 1965; Kirby et al., 1972; White, Roberts, 1983; Aldebert, Traverse, 1984; Goto et al., 1989; Saxena, Shen, 1992].

**Рис. 2.** Рассчитанный коэффициент термического расширения  $\alpha\text{-Al}_2\text{O}_3$  (линия) в сравнении со справочными и экспериментальными данными [Wachtman et al., 1962; Schauer, 1965; Kirby et al., 1972; White, Roberts, 1983; Aldebert, Traverse, 1984; Goto et al., 1989; Saxena, Shen, 1992].



**Fig. 3.** Calculated isothermal ( $K_T$ ) and adiabatic ( $K_S$ ) bulk moduli of  $\alpha\text{-Al}_2\text{O}_3$  in comparison with experimental data [Chung, Simmons, 1968; Goto et al., 1989; Anderson, Isaak, 1995].

**Рис. 3.** Рассчитанные изотермический ( $K_T$ ) и адиабатический ( $K_S$ ) модули сжатия  $\alpha\text{-Al}_2\text{O}_3$  в сравнении с экспериментальными данными [Chung, Simmons, 1968; Goto et al., 1989; Anderson, Isaak, 1995].



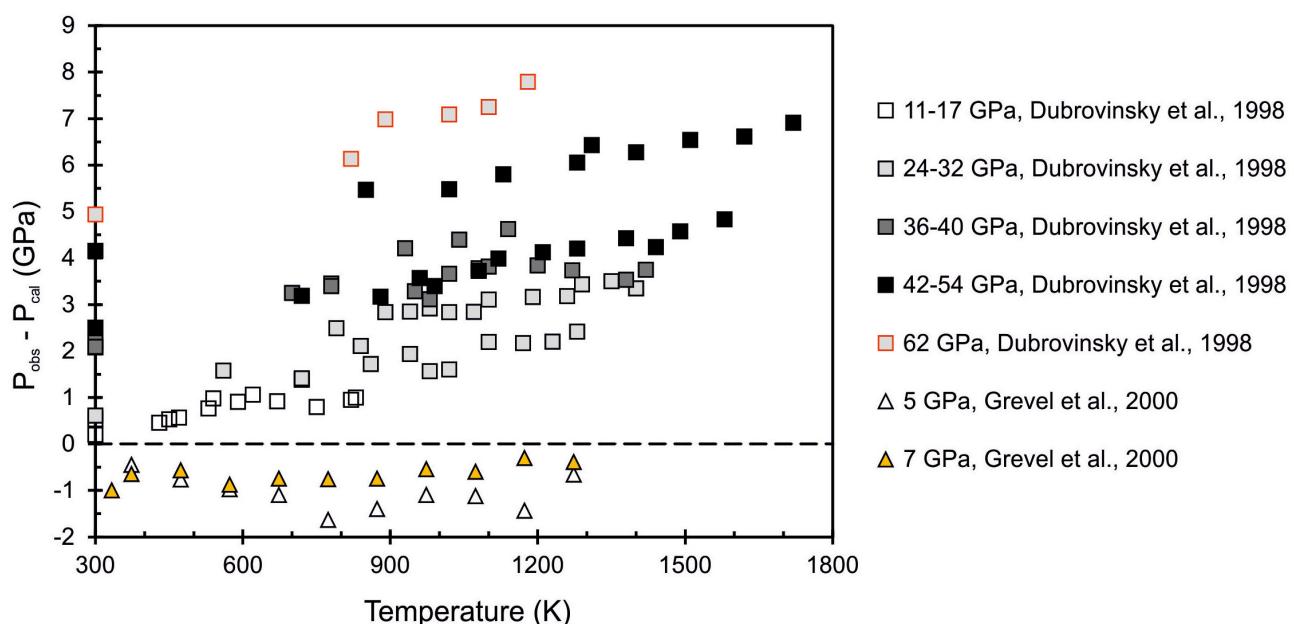
**Fig. 4.** Difference between observed pressure ( $P_{\text{obs}}$ ) [Dewaele, Torrent, 2013; Richet et al., 1988; Kim-Zajonz et al., 1999; Grevel et al., 2000; Dubrovinsky et al., 1998; Jephcoat et al., 1988; Funamori, Jeanloz, 1997; d'Amour et al., 1978; Finger, Hazen, 1978] and calculated pressure ( $P_{\text{cal}}$ ) from equation of state of corundum at 300 K isotherm. The measurements in [Dewaele, Torrent, 2013] are recalculated to the ruby scale from [Dorogokupets et al., 2012; Sokolova et al., 2013]. The upper figure is limited to a pressure 30 GPa.

**Рис. 4.** Разница между измеренным давлением ( $P_{\text{obs}}$ ) [Dewaele, Torrent, 2013; Richet et al., 1988; Kim-Zajonz et al., 1999; Grevel et al., 2000; Dubrovinsky et al., 1998; Jephcoat et al., 1988; Funamori, Jeanloz, 1997; d'Amour et al., 1978; Finger, Hazen, 1978] и рассчитанным давлением ( $P_{\text{cal}}$ ) по уравнению состояния корунда на изотерме 300 К. Измерения из работы [Dewaele, Torrent, 2013] были пересчитаны на основе рубиновой шкалы из работ [Dorogokupets et al., 2012; Sokolova et al., 2013]. Верхний рисунок приведен для области низких давлений до 30 ГПа.

making it difficult to precisely measure. Magnetic transitions for  $\alpha\text{-Fe}_2\text{O}_3$  and  $\text{Fe}_3\text{O}_4$  are calculated at 950 K and 845.5 K, respectively.

The deviations in the pressures calculated from the EoS of eskolaite in this study and observed in [Dymshits et al., 2016] are limited to  $\Delta P/P = \pm 1.5\%$  up to 1873 K (Fig. 9). The difference between pressures does not exceed  $\pm 1$  GPa in the measurements at reference iso-

therm [Sato, Akimoto, 1979; Finger, Hazen, 1980; Kantor et al., 2012]. The difference between the observed pressure and calculated pressure from equations of state of hematite and magnetite at the reference isotherm is  $\Delta P/P = \pm 0.6\%$  (Fig. 10 a, b). Thus, the proposed equations of state of  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$  are in good agreement with the experimental studies of these minerals.



**Fig. 5.** Difference between observed pressure ( $P_{\text{obs}}$ ) [Dubrovinsky et al., 1998; Grevel et al., 2000] and calculated pressure ( $P_{\text{cal}}$ ) from equation of state of corundum at high temperatures.

**Рис. 5.** Разница между измеренным давлением ( $P_{\text{obs}}$ ) [Dubrovinsky et al., 1998; Grevel et al., 2000] и рассчитанным давлением ( $P_{\text{cal}}$ ) по уравнению состояния корунда при высоких температурах.

The calculated thermodynamic functions of  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$  depending on temperature at different pressures are listed in Tables 3–5. The calculated Gibbs energy under standard conditions for  $\text{Cr}_2\text{O}_3$

( $G_{298}=U_0=-1161.25 \text{ kJmol}^{-1}$ ),  $\alpha\text{-Fe}_2\text{O}_3$  ( $G_{298}=U_0=-851.78 \text{ kJmol}^{-1}$ ) and  $\text{Fe}_3\text{O}_4$  ( $G_{298}=U_0=-1158.10 \text{ kJmol}^{-1}$ ) agree well with the results reported in [Holland, Powell, 2011] ( $G^*$  in Tables 3–5).

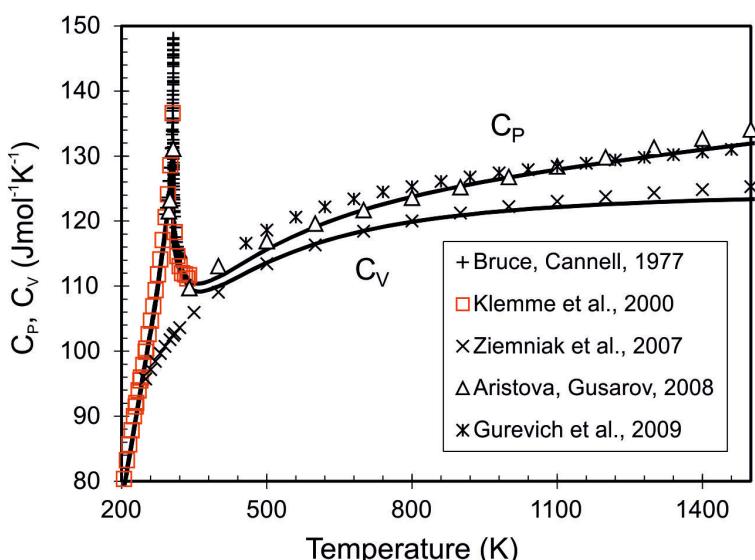
**Table 2. Thermodynamic functions of  $\alpha\text{-Al}_2\text{O}_3$  calculated at different pressures**

**Таблица 2. Рассчитанные термодинамические функции  $\alpha\text{-Al}_2\text{O}_3$  при разных давлениях**

| $P$    | $T$    | $x=V/V_0$ | $\alpha$               | $S$     | $C_P$                           | $C_V$   | $K_T$  | $K_S$  | $\gamma_{\text{th}}$ | $G$      | $G^*$               |
|--------|--------|-----------|------------------------|---------|---------------------------------|---------|--------|--------|----------------------|----------|---------------------|
| ГПа    | К      |           | $10^{-6}\text{K}^{-1}$ |         | $\text{Jmol}^{-1}\text{K}^{-1}$ |         |        | ГПа    |                      |          | $\text{kJmol}^{-1}$ |
| 0.0001 | 298.15 | 1.00000   | 16.223                 | 50.841  | 79.702                          | 79.196  | 252.30 | 253.91 | 1.32                 | -1690.49 | -1690.43            |
| 0.0001 | 500    | 1.00396   | 22.034                 | 99.557  | 106.494                         | 104.956 | 246.82 | 250.44 | 1.33                 | -1705.86 | -1705.85            |
| 0.0001 | 1000   | 1.01659   | 27.170                 | 180.516 | 124.729                         | 120.290 | 231.24 | 239.77 | 1.36                 | -1777.69 | -1777.66            |
| 0.0001 | 1500   | 1.03135   | 30.449                 | 232.521 | 131.778                         | 123.910 | 214.50 | 228.13 | 1.39                 | -1881.70 | -1881.54            |
| 0.0001 | 2000   | 1.04807   | 33.969                 | 271.243 | 137.736                         | 125.561 | 196.82 | 215.90 | 1.43                 | -2008.04 | -2007.76            |
| 50     | 298.15 | 0.86330   | 7.179                  | 38.199  | 68.078                          | 67.928  | 439.98 | 440.94 | 1.03                 | -509.29  | -507.54             |
| 50     | 500    | 0.86488   | 10.458                 | 81.777  | 98.481                          | 97.953  | 436.06 | 438.40 | 1.03                 | -521.51  | -519.25             |
| 50     | 1000   | 0.87009   | 12.976                 | 158.419 | 119.401                         | 117.811 | 424.46 | 430.19 | 1.04                 | -583.17  | -580.00             |
| 50     | 1500   | 0.87599   | 13.967                 | 208.100 | 125.240                         | 122.538 | 412.08 | 421.16 | 1.05                 | -675.54  | -671.10             |
| 50     | 2000   | 0.88230   | 14.721                 | 244.597 | 128.450                         | 124.545 | 399.30 | 411.83 | 1.07                 | -789.12  | -784.89             |
| 80     | 298.15 | 0.81198   | 5.245                  | 34.264  | 63.917                          | 63.824  | 541.42 | 542.20 | 0.92                 | 132.51   | 136.95              |
| 80     | 500    | 0.81308   | 7.878                  | 75.950  | 95.500                          | 95.153  | 537.97 | 539.94 | 0.93                 | 121.30   | 126.61              |
| 80     | 1000   | 0.81680   | 9.894                  | 151.108 | 117.861                         | 116.782 | 527.56 | 532.43 | 0.93                 | 63.01    | 69.89               |
| 80     | 1500   | 0.82101   | 10.598                 | 200.205 | 123.809                         | 121.982 | 516.39 | 524.13 | 0.94                 | -25.55   | -17.46              |
| 80     | 2000   | 0.82548   | 11.076                 | 236.262 | 126.766                         | 124.151 | 504.90 | 515.54 | 0.95                 | -135.06  | -126.12             |

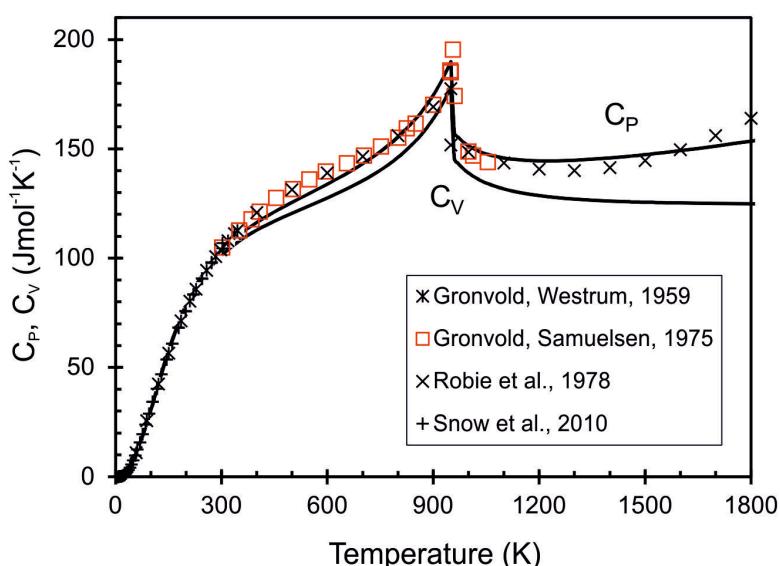
**Note.**  $G$  – data calculated in this study;  $G^*$  – data from [Holland, Powell, 2011].

**Примечание.**  $G$  – данные, рассчитанные в настоящей работе;  $G^*$  – данные из работы [Holland, Powell, 2011].



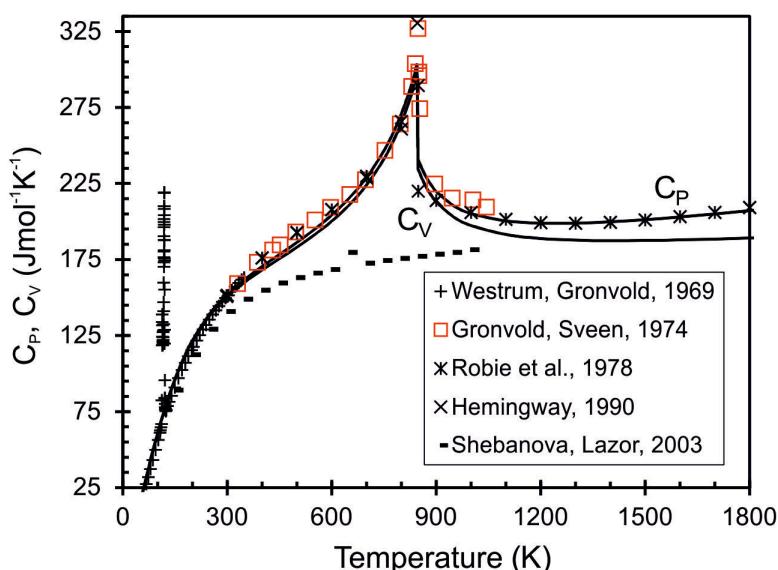
**Fig. 6.** Calculated isobaric and isochoric heat capacity of  $\text{Cr}_2\text{O}_3$  (solid lines) in comparison with selected reference and experimental data [Bruce, Cannell, 1977; Klemme et al., 2000; Ziemniak et al., 2007; Aristova, Gusalov, 2008; Gurevich et al., 2009].

**Рис. 6.** Рассчитанная изобарная и изохорная теплоемкость  $\text{Cr}_2\text{O}_3$  (линии) в сравнении со справочными и экспериментальными данными [Bruce, Cannell, 1977; Klemme et al., 2000; Ziemniak et al., 2007; Aristova, Gusalov, 2008; Gurevich et al., 2009].



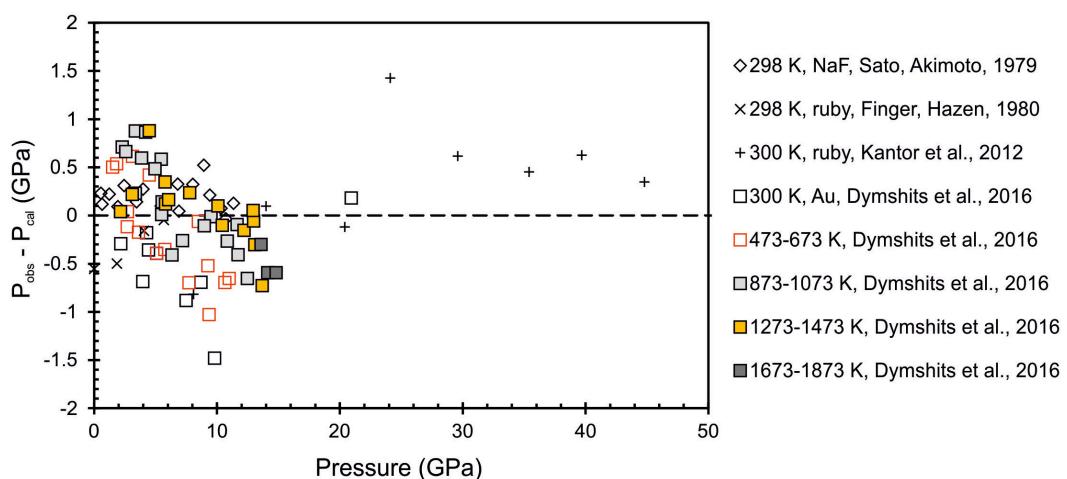
**Fig. 7.** Calculated isobaric and isochoric heat capacity of  $\alpha\text{-Fe}_2\text{O}_3$  (solid lines) in comparison with selected reference and experimental data [Gronvold, Westrum, 1959; Gronvold, Samuelsen, 1975; Robie et al., 1978; Snow et al., 2010].

**Рис. 7.** Рассчитанная изобарная и изохорная теплоемкость  $\alpha\text{-Fe}_2\text{O}_3$  (линии) в сравнении со справочными и экспериментальными данными [Gronvold, Westrum, 1959; Gronvold, Samuelsen, 1975; Robie et al., 1978; Snow et al., 2010].



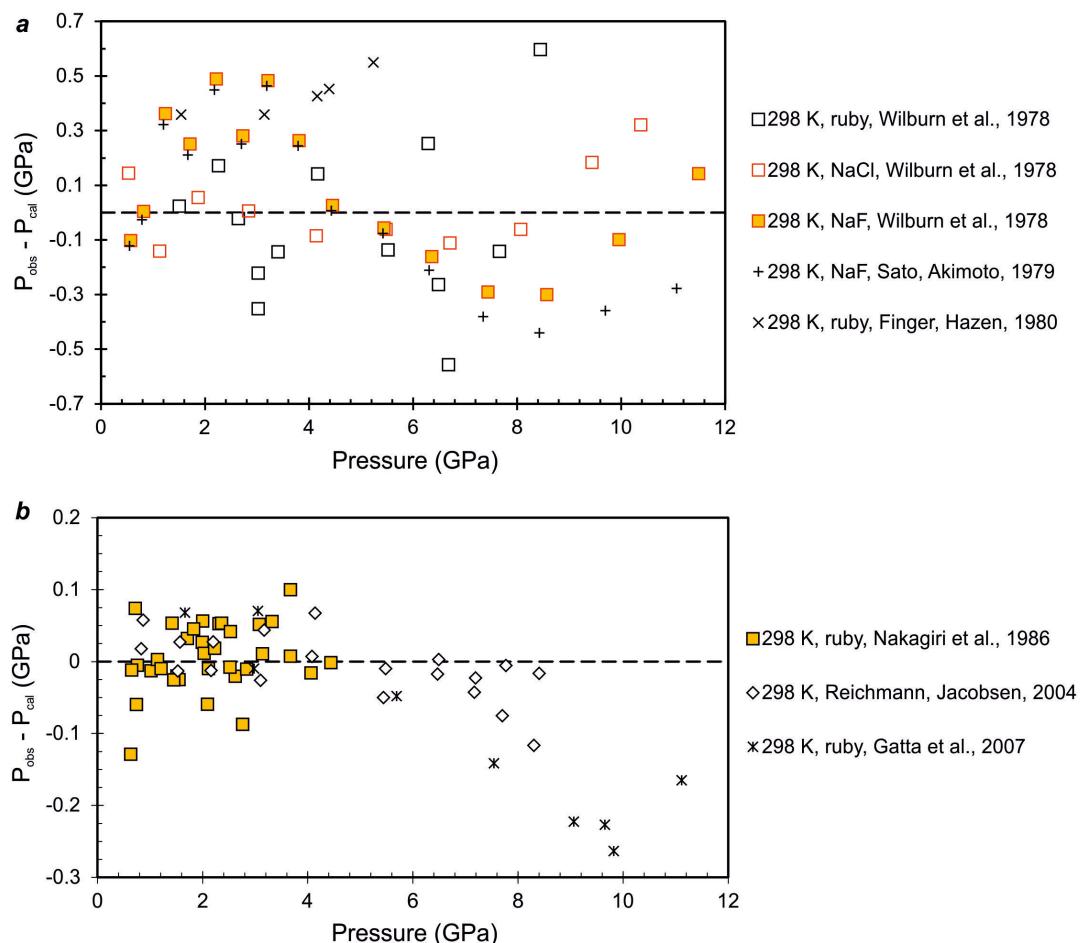
**Fig. 8.** Calculated isobaric and isochoric heat capacity of  $\text{Fe}_3\text{O}_4$  (solid lines) in comparison with selected reference and experimental data [Westrum, Gronvold, 1969; Gronvold, Sveen, 1974; Robie et al., 1978; Hemingway, 1990; Shebanova, Lazor, 2003].

**Рис. 8.** Рассчитанная изобарная и изохорная теплоемкость  $\text{Fe}_3\text{O}_4$  (линии) в сравнении со справочными и экспериментальными данными [Westrum, Gronvold, 1969; Gronvold, Sveen, 1974; Robie et al., 1978; Hemingway, 1990; Shebanova, Lazor, 2003].



**Fig. 9.** Difference between observed pressure in [Sato, Akimoto, 1979; Finger, Hazen, 1980; Kantor et al., 2012; Dymshits et al., 2016] and calculated pressure from equation of state of eskolaite at the isotherms 298 to 1873 K.

**Рис. 9.** Разница между измеренным давлением в работах [Sato, Akimoto, 1979; Finger, Hazen, 1980; Kantor et al., 2012; Dymshits et al., 2016] и рассчитанным давлением по уравнению состояния эсколаита на изотермах от 298 до 1873 К.



**Fig. 10.** Difference between observed pressure in [Wilburn et al., 1978; Sato, Akimoto, 1979; Finger, Hazen, 1980; Nakagiri et al., 1986; Reichmann, Jacobsen, 2004; Gatta et al., 2007] and calculated pressure from equation of state of hematite (a) and magnetite (b) at the reference isotherm 298 K.

**Рис. 10.** Разница между измеренным давлением в работах [Wilburn et al., 1978; Sato, Akimoto, 1979; Finger, Hazen, 1980; Nakagiri et al., 1986; Reichmann, Jacobsen, 2004; Gatta et al., 2007] и рассчитанным по уравнениям состояния гематита (a) и магнетита (b) на отсчетной изотерме 298 К.

Table 3. Thermodynamic functions of Cr<sub>2</sub>O<sub>3</sub> calculated at different pressuresТаблица 3. Рассчитанные термодинамические функции Cr<sub>2</sub>O<sub>3</sub> при разных давлениях

| P      | T      | x=V/V <sub>0</sub> | $\alpha$ | S                                | C <sub>P</sub>                     | C <sub>V</sub> | K <sub>T</sub> | K <sub>S</sub> | $\gamma_{th}$ | G        | G*                  |
|--------|--------|--------------------|----------|----------------------------------|------------------------------------|----------------|----------------|----------------|---------------|----------|---------------------|
| ГПа    | К      |                    |          | 10 <sup>-6</sup> К <sup>-1</sup> | Jmol <sup>-1</sup> К <sup>-1</sup> |                |                | ГПа            |               |          | kJmol <sup>-1</sup> |
| 0.0001 | 298.15 | 1                  | 21.211   | 80.2160                          | 123.793                            | 122.968        | 211.70         | 213.12         | 1.06          | -1161.25 | -1162.08            |
| 0.0001 | 500    | 1.0049             | 25.942   | 138.785                          | 115.587                            | 113.560        | 206.32         | 210.00         | 1.38          | -1183.87 | -1185.12            |
| 0.0001 | 1000   | 1.0191             | 29.951   | 223.073                          | 126.685                            | 121.584        | 192.02         | 200.08         | 1.40          | -1276.44 | -1279.12            |
| 0.0001 | 1500   | 1.0352             | 32.669   | 275.479                          | 131.894                            | 123.358        | 177.27         | 189.53         | 1.41          | -1401.87 | -1406.11            |
| 0.0001 | 2000   | 1.0530             | 35.586   | 314.057                          | 136.553                            | 123.984        | 162.18         | 178.62         | 1.42          | -1549.66 | -1555.56            |
| 30     | 298.15 | 0.8986             | 12.133   | 67.1730                          | 115.728                            | 115.317        | 358.64         | 359.92         | 0.99          | -338.36  | -335.46             |
| 30     | 500    | 0.9012             | 15.427   | 122.491                          | 110.858                            | 109.755        | 353.95         | 357.51         | 1.30          | -357.97  | -356.14             |
| 30     | 1000   | 0.9088             | 17.672   | 204.185                          | 123.270                            | 120.454        | 341.43         | 349.41         | 1.32          | -441.63  | -443.61             |
| 30     | 1500   | 0.9171             | 18.642   | 255.037                          | 127.392                            | 122.827        | 328.63         | 340.84         | 1.33          | -557.22  | -563.66             |
| 30     | 2000   | 0.9259             | 19.446   | 292.070                          | 130.097                            | 123.673        | 315.72         | 332.12         | 1.34          | -694.41  | -705.81             |
| 70     | 298.15 | 0.8207             | 7.8670   | 57.5000                          | 108.557                            | 108.323        | 531.94         | 533.09         | 0.92          | 657.16   | 663.17              |
| 70     | 500    | 0.8223             | 10.459   | 109.899                          | 106.693                            | 106.004        | 527.65         | 531.08         | 1.24          | 639.85   | 644.49              |
| 70     | 1000   | 0.8270             | 12.141   | 189.579                          | 121.136                            | 119.308        | 516.02         | 523.93         | 1.26          | 563.11   | 562.46              |
| 70     | 1500   | 0.8322             | 12.702   | 239.588                          | 125.241                            | 122.291        | 504.14         | 516.30         | 1.27          | 455.04   | 448.14              |
| 70     | 2000   | 0.8376             | 13.083   | 275.943                          | 127.464                            | 123.364        | 492.18         | 508.53         | 1.27          | 325.75   | 311.88              |

## 5. DISCUSSION AND CONCLUSION

In this study, the equations of state of rock-forming oxides ( $\alpha$ -Al<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub>) are developed based on optimization of variety experimental measurements and can be used for calculation of different thermodynamic properties in a wide range of pressures and temperatures. The proposed approach for constructing of equation of state perfectly approximates the thermochemical data of the heat capacity and allows one to describe the  $\lambda$ -type anomaly in case of changes in magnetic properties of Cr<sub>2</sub>O<sub>3</sub>,  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>, and Fe<sub>3</sub>O<sub>4</sub>.

Nonetheless, other studies concerning equations of state (especially for corundum) are noteworthy. In [Dubrovinskaya et al., 1997], the equation of state is based on the Helmholtz free energy with a single set of common parameters and the Birch-Murnaghan equation on zero isotherm. In this study, the extrapolations of the heat capacity, thermal expansion, elastic moduli and P-V-T properties to high pressures and temperatures are in good agreement with the experimental data. In the computational study reported in [Dorogokupets et al., 1999], an empirical model is constructed for the joint optimization of data on isobaric heat capacity, volume, thermal expansion coefficient, and

Table 4. Thermodynamic functions of  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> calculated at different pressuresТаблица 4. Рассчитанные термодинамические функции  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub> при разных давлениях

| P      | T      | x=V/V <sub>0</sub> | $\alpha$ | S                                | C <sub>P</sub>                     | C <sub>V</sub> | K <sub>T</sub> | K <sub>S</sub> | $\gamma_{th}$ | G        | G*                  |
|--------|--------|--------------------|----------|----------------------------------|------------------------------------|----------------|----------------|----------------|---------------|----------|---------------------|
| ГПа    | К      |                    |          | 10 <sup>-6</sup> К <sup>-1</sup> | Jmol <sup>-1</sup> К <sup>-1</sup> |                |                | ГПа            |               |          | kJmol <sup>-1</sup> |
| 0.0001 | 298.15 | 1                  | 33.873   | 87.695                           | 102.870                            | 100.773        | 202.50         | 206.71         | 2.06          | -851.78  | -851.65             |
| 0.0001 | 500    | 1.0076             | 40.449   | 146.916                          | 125.411                            | 120.511        | 196.37         | 204.35         | 2.01          | -875.78  | -875.81             |
| 0.0001 | 1000   | 1.0301             | 46.982   | 248.732                          | 151.340                            | 138.892        | 180.85         | 197.05         | 1.91          | -975.50  | -977.81             |
| 0.0001 | 1500   | 1.0560             | 52.498   | 307.882                          | 147.287                            | 125.514        | 164.74         | 193.32         | 2.20          | -1116.21 | -1119.53            |
| 0.0001 | 2000   | 1.0858             | 59.146   | 351.684                          | 158.678                            | 124.668        | 147.88         | 188.22         | 2.31          | -1281.57 | -1285.98            |
| 20     | 298.15 | 0.9175             | 23.348   | 71.4180                          | 93.8630                            | 92.673         | 263.70         | 267.09         | 1.85          | -272.74  | -270.04             |
| 20     | 500    | 0.9224             | 29.062   | 126.977                          | 119.836                            | 116.791        | 258.25         | 264.98         | 1.79          | -293.02  | -291.12             |
| 20     | 1000   | 0.9372             | 33.595   | 225.228                          | 145.669                            | 137.842        | 244.42         | 258.30         | 1.69          | -381.78  | -384.52             |
| 20     | 1500   | 0.9537             | 36.394   | 281.513                          | 138.242                            | 125.032        | 230.30         | 254.63         | 1.94          | -510.03  | -516.75             |
| 20     | 2000   | 0.9719             | 39.250   | 321.999                          | 143.954                            | 124.391        | 215.79         | 249.73         | 2.00          | -661.41  | -672.81             |
| 50     | 298.15 | 0.8314             | 15.352   | 56.4180                          | 83.2700                            | 82.6550        | 347.78         | 350.36         | 1.63          | 518.99   | 534.37              |
| 50     | 500    | 0.8344             | 20.335   | 107.672                          | 113.547                            | 111.755        | 342.98         | 348.48         | 1.58          | 502.25   | 516.41              |
| 50     | 1000   | 0.8439             | 23.846   | 202.484                          | 141.166                            | 136.363        | 330.66         | 342.31         | 1.48          | 424.15   | 431.64              |
| 50     | 1500   | 0.8544             | 25.431   | 256.725                          | 132.330                            | 124.347        | 318.15         | 338.57         | 1.68          | 307.79   | 308.70              |
| 50     | 2000   | 0.8656             | 26.790   | 295.184                          | 135.486                            | 123.997        | 305.43         | 333.73         | 1.73          | 169.30   | 162.50              |

Table 5. Thermodynamic functions of  $\text{Fe}_3\text{O}_4$  calculated at different pressuresТаблица 5. Рассчитанные термодинамические функции  $\text{Fe}_3\text{O}_4$  при разных давлениях

| $P$    | $T$    | $x=V/V_0$ | $\alpha$ | $S$                    | $C_P$                           | $C_V$   | $K_T$  | $K_S$  | $\gamma_{\text{th}}$ | $G$      | $G^*$               |
|--------|--------|-----------|----------|------------------------|---------------------------------|---------|--------|--------|----------------------|----------|---------------------|
| ГПа    | К      |           |          | $10^{-6}\text{K}^{-1}$ | $\text{Jmol}^{-1}\text{K}^{-1}$ |         |        | ГПа    |                      |          | $\text{kJmol}^{-1}$ |
| 0.0001 | 298.15 | 1         | 24.251   | 146.246                | 151.503                         | 150.087 | 181.20 | 182.91 | 1.31                 | -1158.10 | -1158.27            |
| 0.0001 | 500    | 1.0054    | 28.235   | 233.239                | 186.151                         | 183.001 | 176.33 | 179.37 | 1.22                 | -1196.85 | -1197.39            |
| 0.0001 | 1000   | 1.0209    | 32.553   | 387.280                | 205.181                         | 197.300 | 163.43 | 169.96 | 1.23                 | -1355.10 | -1357.68            |
| 0.0001 | 1500   | 1.0386    | 36.341   | 468.424                | 201.166                         | 187.424 | 149.83 | 160.82 | 1.35                 | -1570.44 | -1575.12            |
| 0.0001 | 2000   | 1.0588    | 40.832   | 527.662                | 211.941                         | 190.604 | 135.56 | 150.74 | 1.37                 | -1820.04 | -1827.81            |
| 10     | 298.15 | 0.9522    | 18.916   | 136.960                | 147.615                         | 146.581 | 228.34 | 229.95 | 1.25                 | -723.45  | -724.10             |
| 10     | 500    | 0.9562    | 22.177   | 222.341                | 183.605                         | 181.259 | 223.82 | 226.72 | 1.17                 | -760.15  | -760.04             |
| 10     | 1000   | 0.9677    | 25.214   | 374.643                | 202.257                         | 196.446 | 211.88 | 218.15 | 1.17                 | -912.48  | -911.40             |
| 10     | 1500   | 0.9805    | 27.470   | 454.289                | 196.413                         | 186.547 | 199.41 | 209.95 | 1.28                 | -1121.13 | -1118.73            |
| 10     | 2000   | 0.9947    | 29.893   | 511.790                | 204.276                         | 189.499 | 186.46 | 201.00 | 1.30                 | -1363.24 | -1359.93            |
| 20     | 298.15 | 0.9149    | 15.603   | 129.825                | 144.433                         | 143.626 | 272.65 | 274.18 | 1.21                 | -307.59  | -308.91             |
| 20     | 500    | 0.9181    | 18.440   | 213.912                | 181.640                         | 179.772 | 268.38 | 271.16 | 1.13                 | -342.69  | -342.29             |
| 20     | 1000   | 0.9272    | 20.849   | 364.985                | 200.368                         | 195.749 | 257.06 | 263.13 | 1.13                 | -490.46  | -486.60             |
| 20     | 1500   | 0.9373    | 22.435   | 443.709                | 193.608                         | 185.870 | 245.28 | 255.49 | 1.24                 | -694.05  | -686.11             |
| 20     | 2000   | 0.9482    | 24.036   | 500.223                | 200.070                         | 188.684 | 233.12 | 247.19 | 1.26                 | -930.63  | -918.69             |

bulk moduli of minerals at room pressure. The internal energy and isochoric heat capacity in this study are approximated by the Nernst-Lindeman function. The comparison of the Nernst-Lindeman function and our model with two characteristic temperatures (eq. 3) shows that the approximations look almost identical, so in both cases they provide good smoothing and approximation of the heat capacity, thermal expansion coefficient, and bulk moduli at reference pressure. Besides, other EoSes are available to calculate thermodynamic properties of corundum at high temperatures and pressures, such as described in [Jacobs, Oonk, 2001; Jacobs, Shmid-Fetzer, 2010; Jacobs et al., 2013; Stixrude, Lithgow-Bertelloni, 2005; Brosh et al., 2008; Otero-de-la-Roza, Luana, 2011].

The formalism in studies by Gerya et al. [1998, 2004] is considered thermodynamics of minerals based on the Gibbs energy. It is of special interest and may be used to construct the equations of state of corundum and phases with  $\lambda$ -type anomaly in the heat capacity, including  $\text{Cr}_2\text{O}_3$ ,  $\alpha\text{-Fe}_2\text{O}_3$ , and  $\text{Fe}_3\text{O}_4$ .

The thermodynamic properties of rock-forming oxides, which are established in this study, can provide a very useful contribution for geobarometry and modeling of the mineral composition of the Earth. The presence of sesquioxide stabilizes in the solid-state systems at depth and plays an important role in understanding the mantle mineralogy and structural transformations, which associated with the global and intermediate boundaries in the Earth's mantle (olivine  $\rightarrow$  wadsleyite  $\rightarrow$  ringwoodite; kyanite  $\rightarrow$  corundum + stishovite; garnet  $\rightarrow$  perovskite + corundum-ilmenite) [Pushcharovsky, Pushcharovsky, 2010]. Modeling of solid solutions based on iron oxide can be highly important for describing pos-

sible associations in the Earth's core and its solubility in the mantle phases. Chromium oxide with small concentrations of iron and aluminum was discovered as inclusions in diamond from the Udachnaya kimberlite pipe, Yakutia [Logvinova et al., 2008]. Because correct thermal equation of state of this phase is lacking it is impossible to calculate the released pressure of such inclusions.

The rock-forming oxides may form solid solutions with spinel structure. These spinel phases may contain variable amounts of trivalent cations  $\text{Fe}^{3+}$ ,  $\text{Al}^{3+}$  and  $\text{Cr}^{3+}$  and divalent cations  $\text{Fe}^{2+}$  and  $\text{Mg}^{2+}$ . The properties of such materials are interesting and important for not only geology and geophysics. The refractoriness of some of these oxides is recognized as an extremely valuable characteristic in certain special applications in steelmaking furnace industry. Therefore, the temperature dependence of elastic and thermodynamic parameters, such as cell volume, bulk modulus, thermal expansion and the Grüneisen parameter of oxides, are significant for understanding the material dynamics with respect to thermal and pressure stress. The phase relations of the  $\text{Cr}_2\text{O}_3\text{-Fe}_2\text{O}_3\text{-Al}_2\text{O}_3$  system at reference pressure have been known for more than 50 years [Muan, Somiya, 1959; Schultz, Stubican, 1970], but the knowledge of high-pressure behavior is still lacking.

The thermodynamic parameters of corundum, eskolaite, hematite, and magnetite, which are calculated in this study, specify the thermodynamics of pure oxide phases according to the current  $P\text{-}V\text{-}T$  measurements obtained in diamond anvils and multianvil apparatus [Dewaele, Torrent, 2013; Dymshits et al., 2016], and can be applied in calculations of more complex mineral systems and solid solutions at high temperatures and pressures.

## 6. ACKNOWLEDGMENTS

We thanks PhD B.S. Danilov for his help in calculations of the Gibbs energy from the database [Holland, Powell, 2011]. This study was supported by the Russian

Scientific Foundation (Project no 14-17-00601) and the Russian Foundation of Basic Research (Projects no. 15-35-20556 and 16-35-00061) and conducted under the program of the Ministry of Education and Science of the Russian Federation (Project no 14.B.25.31.0032).

## 7. ЛИТЕРАТУРА / REFERENCES

- Aldebert P., Traverse J.P., 1984.  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>: A high temperature thermal expansion standard. *High Temperatures. High Pressures* 16 (2), 127–135.
- Al'tshuler L.V., Brusnikin S.E., Kuz'menkov E.A., 1987. Isotherms and Grüneisen functions for 25 metals. *Journal of Applied Mechanics and Technical Physics* 28 (1), 129–141. <http://dx.doi.org/10.1007/BF00918785>.
- d'Amour H., Schiferl D., Denner W., Schulz H., Holzapfel W.B., 1978. High-pressure single-crystal structure determinations for ruby up to 90 kbar using an automatic diffractometer. *Journal of Applied Physics* 49 (8), 4411–4416. <http://dx.doi.org/10.1063/1.325494>.
- Anderson O.L., Isaak D.G., 1995. Elastic constants of mantle minerals at high temperature. In: T.J. Ahrens (Ed.), Mineral physics and crystallography. A Handbook of Physical Constants. AGU Reference Shelf 2. AGU, Washington, p. 64–97. <http://dx.doi.org/10.1029/RF002p0064>.
- Archer D.G., 1993. Thermodynamic properties of synthetic sapphire ( $\alpha$ -Al<sub>2</sub>O<sub>3</sub>), standard reference material 720 and the effect of temperature-scale differences on thermodynamic properties. *Journal of Physical and Chemical Reference Data* 22 (6), 1441–1453. <http://dx.doi.org/10.1063/1.555931>.
- Aristova N.M., Gusarov A.V., 2008. ChemNet. Available from: <http://www.chem.msu.su/Zn/Cr/welcome.html> (last accessed July 14, 2016).
- Brosh E., Shneck Z., Makov G., 2008. Explicit Gibbs free energy equation of state for solids. *Journal of Physics and Chemistry of Solids* 69 (8), 1912–1922. <http://dx.doi.org/10.1016/j.jpcs.2008.01.019>.
- Brown M.J., 1999. The NaCl pressure standard. *Journal of Applied Physics* 86 (10), 5801–5808. <http://dx.doi.org/10.1063/1.371596>.
- Bruce R.H., Cannel D.S., 1977. Specific heat of Cr<sub>2</sub>O<sub>3</sub> near the Neel temperature. *Physical Review B* 15 (9), 4451–4458. <http://dx.doi.org/10.1103/PhysRevB.15.4451>.
- Chase M.W., 1998. NIST-JANAF thermochemical tables. Fourth Edition. *Journal of Physical and Chemical Reference Data*, Monograph 9. AIP, New York, 1951 p.
- Chung D.H., Simmons G., 1968. Pressure and temperature dependences of the isotropic elastic moduli of polycrystalline alumina. *Journal of Applied Physics* 39 (11), 5316–5326. <http://dx.doi.org/10.1063/1.1655961>.
- Decker D.L., 1971. High-pressure equation of state for NaCl, KCl, and CsCl. *Journal of Applied Physics* 42 (8), 3239–3244. <http://dx.doi.org/10.1063/1.1660714>.
- Dera P., Lavina B., Meng Y., Prakapenka V.B., 2011. Structural and electronic evolution of Cr<sub>2</sub>O<sub>3</sub> on compression to 55 GPa. *Journal of Solid State Chemistry* 184 (11), 3040–3049. <http://dx.doi.org/10.1016/j.jssc.2011.09.021>.
- Dewaele A., Torrent M., 2013. Equation of state of  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>. *Physical Review B* 88 (6), 064107. <http://dx.doi.org/10.1103/PhysRevB.88.064107>.
- Dinsdale A.T., 1991. SGTE data for pure elements. *CALPHAD* 15 (4), 317–425. [http://dx.doi.org/10.1016/0364-5916\(91\)90030-N](http://dx.doi.org/10.1016/0364-5916(91)90030-N).
- Dorogokupets P.I., 1995. Equation of state for lambda transition in quartz. *Journal of Geophysical Research* 100 (B5), 8489–8499. <http://dx.doi.org/10.1029/94JB02917>.
- Dorogokupets P.I., 2010. P-V-T equations of state of MgO and thermodynamics. *Physics and Chemistry of Minerals* 37 (9), 677–684. <http://dx.doi.org/10.1007/s00269-010-0367-2>.
- Dorogokupets P.I., Dewaele A., 2007. Equations of state of MgO, Au, Pt, NaCl-B1, and NaCl-B2: Internally consistent high-temperature pressure scales. *High Pressure Research* 27 (4), 431–446. <http://dx.doi.org/10.1080/08957950701659700>.
- Dorogokupets P.I., Dymshits A.M., Sokolova T.S., Danilov B.S., Litasov K.D., 2015. The equations of state of forsterite, wadsleyite, ringwoodite, akimotoite, MgSiO<sub>3</sub>-perovskite, and postperovskite and phase diagram for the Mg<sub>2</sub>SiO<sub>4</sub> system at pressures of up to 130 GPa. *Russian Geology and Geophysics* 56 (1–2), 172–189. <http://dx.doi.org/10.1016/j.rgg.2015.01.011>.
- Dorogokupets P.I., Oganov A.R., 2007. Ruby, metals, and MgO as alternative pressure scales: A semiempirical description of shock-wave, ultrasonic, x-ray, and thermochemical data at high temperatures and pressures. *Physical Review B* 75 (2), 024115. <http://dx.doi.org/10.1103/PhysRevB.75.024115>.

Dorogokupets P.I., Ponomarev E.M., Melekhova E.A., 1999. Optimization of experimental data on the heat capacity, volume, and bulk moduli of minerals. *Petrology* 7 (6), 574–591.

Dorogokupets P.I., Sokolova T.S., Danilov B.S., Litasov K.D., 2012. Near-absolute equations of state of diamond, Ag, Al, Au, Cu, Mo, Nb, Pt, Ta, and W for quasi-hydrostatic conditions. *Geodynamics and Tectonophysics* 3 (2), 129–166 (in Russian) [Дорогокупец П.И., Соколова Т.С., Данилов Б.С., Литасов К.Д. Почти абсолютные уравнения состояния алмаза, Ag, Al, Au, Cu, Mo, Nb, Pt, Ta, W для квазигидростатических условий // Геодинамика и тектонофизика. 2012. Т. 3. № 2. С. 129–166]. <http://dx.doi.org/10.5800/GT-2012-3-2-0067>.

Dorogokupets P.I., Sokolova T.S., Litasov K.D., 2014. Thermodynamic properties of bcc-Fe to melting temperature and pressure to 15 GPa. *Geodynamics and Tectonophysics* 5 (4), 1033–1044 (in Russian) [Дорогокупец П.И., Соколова Т.С., Литасов К.Д. Термодинамические свойства бсс-Фе до температуры плавления и до давления 15 ГПа // Геодинамика и тектонофизика. 2014. Т. 5. № 4. С. 1033–1044]. <http://dx.doi.org/10.5800/GT-2014-5-4-0166>.

Dubrovinskaya N.A., Dubrovinsky L.S., Saxena S.K., 1997. Systematics of thermodynamic data on solids: Thermochemical and pressure-volume-temperature properties of some minerals. *Geochimica et Cosmochimica Acta* 61 (19), 4151–4158. [http://dx.doi.org/10.1016/S0016-7037\(97\)00233-0](http://dx.doi.org/10.1016/S0016-7037(97)00233-0).

Dubrovinsky L.S., Saxena S.K., Lazor P., 1998. High-pressure and high-temperature in situ X-ray diffraction study of iron and corundum to 68 GPa using an internally heated diamond anvil cell. *Physical and Chemistry of Minerals* 25 (6), 434–441. <http://dx.doi.org/10.1007/s002690050133>.

Dymshits A.M., Dorogokupets P.I., Sharygin I.S., Litasov K.D., Shatskiy A., Rashchenko S.V., Ohtani E., Suzuki A., Higo Y., 2016. Thermoelastic properties of chromium oxide Cr<sub>2</sub>O<sub>3</sub> (eskolaite) at high pressures and temperatures. *Physics and Chemistry of Minerals* 43 (6), 447–458. <http://dx.doi.org/10.1007/s00269-016-0808-7>.

Finger L.W., Hazen R.M., 1978. Crystal structure and compression of ruby to 46 kbar. *Journal of Applied Physics* 49 (12), 5823–5826. <http://dx.doi.org/10.1063/1.324598>.

Finger L.W., Hazen R.M., 1980. Crustal structure and isothermal compression of Fe<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>, and V<sub>2</sub>O<sub>3</sub> to 50 kbars. *Journal of Applied Physics* 51 (10), 5362–5367. <http://dx.doi.org/10.1063/1.327451>.

Funamori N., Jeanloz R., 1997. High-pressure transformation of Al<sub>2</sub>O<sub>3</sub>. *Science* 278 (5340), 1109–1111. <http://dx.doi.org/10.1126/science.278.5340.1109>.

Gatta G.D., Kantor I., Ballaran T.B., Dubrovinsky L., McCammon C., 2007. Effect of non-hydrostatic conditions on the elastic behavior of magnetite: an in situ single-crystal X-ray diffraction study. *Physical and Chemistry of Minerals* 34 (9), 627–635. <http://dx.doi.org/10.1007/s00269-007-0177-3>.

Gerya T.V., Maresch W.V., Podlesskii K.K., Perchuk L.L., 2004. Semi-empirical Gibbs free energy formulations for minerals and fluids for use in thermodynamic databases of petrological interest. *Physics and Chemistry of Minerals* 31 (7), 429–455. <http://dx.doi.org/10.1007/s00269-004-0409-8>.

Gerya T.V., Podlesskii K.K., Perchuk L.L., Swamy V., Kosyakova N.A., 1998. Equations of state of minerals for thermodynamic databases used in petrology. *Petrology* 6 (6), 511–526.

Goto T., Anderson O.L., Ohno I., Yamamoto S., 1989. Elastic constants of corundum up to 1825 K. *Journal of Geophysical Research* 94 (B6), 7588–7602. <http://dx.doi.org/10.1029/JB094iB06p07588>.

Grevel K.-D., Burchard M., Fabhauer D.F., 2000. Pressure-volume-temperature behavior of diaspore and corundum: An in situ X-ray diffraction study comparing different pressure media. *Journal of Geophysical Research* 105 (B12), 27877–27887. <http://dx.doi.org/10.1029/2000JB900323>.

Gronvold F., Samuels E.J., 1975. Heat capacity and thermodynamic properties of α-Fe<sub>2</sub>O<sub>3</sub> in the region 300–1050 K. Antiferromagnetic transition. *Journal of Physics and Chemistry Solids* 36 (4), 249–256. [http://dx.doi.org/10.1016/0022-3697\(75\)90017-7](http://dx.doi.org/10.1016/0022-3697(75)90017-7).

Gronvold F., Sveen A., 1974. Heat capacity and thermodynamic properties of synthetic magnetite (Fe<sub>3</sub>O<sub>4</sub>) from 300 to 1050 K. Ferrimagnetic transition and zero-point entropy. *Journal of Chemistry Thermodynamics* 6 (9), 859–872. [http://dx.doi.org/10.1016/0021-9614\(74\)90230-4](http://dx.doi.org/10.1016/0021-9614(74)90230-4).

Gronvold F., Westrum E.F., 1959. α-Ferric Oxide: low temperature heat capacity and thermodynamic functions. *Journal of the American Chemical Society* 81 (8), 1780–1783. <http://dx.doi.org/10.1021/ja01517a002>.

Gurevich V.M., Kuskov O.L., Smirnova N.N., Gavrichev K.S., Markin A.V., 2009. Thermodynamic functions of eskolaite Cr<sub>2</sub>O<sub>3</sub>(c) at 0–1800 K. *Geochemistry International* 47 (12), 1170–1179. <http://dx.doi.org/10.1134/S0016702909120027>.

Gurvich L.V., Veits I.V., Medvedev V.A., 1981. Thermodynamic Properties of Substances. Vol. 3. Nauka, Moscow, 400 p. (in Russian) [Гурвич Л.В., Вейц И.В., Медведев В.А. Термодинамические свойства индивидуальных веществ. М.: Наука, 1981. Т. 3. 400 с.]

Hemingway B.S., 1990. Thermodynamic properties for bunsenite, NiO, magnetite, Fe<sub>3</sub>O<sub>4</sub>, and hematite, Fe<sub>2</sub>O<sub>3</sub> with comments on selected oxygen buffer reactions. *American Mineralogist* 75 (7–8), 781–790.

Hill A.H., Harrison A., Dickinson C., Zhou W., Kockelmann W., 2010. Crystallographic and magnetic studies of mesoporous eskolaite, Cr<sub>2</sub>O<sub>3</sub>. *Microporous and Mesoporous Materials* 130 (1–3), 280–286. <http://dx.doi.org/10.1016/j.micromeso.2009.11.021>.

- Holland T.J.B., Powell R., 2011. An improved and extended internally consistent thermodynamic dataset for phases of petrological interest, involving a new equation of state for solids. *Journal of Metamorphic Geology* 29 (3), 333–383. <http://dx.doi.org/10.1111/j.1525-1314.2010.00923.x>.
- Ito E., Fukui H., Katsura T., Yamazaki D., Yoshino T., Aizawa Y., Kubo A., Yokoshi S., Kawabe K., Zhai S., Shatzkiy A., Okube M., Nozawa A., Funakoshi K.-I., 2009. Determination of high-pressure phase equilibria of Fe<sub>2</sub>O<sub>3</sub> using the Kawai-type apparatus equipped with sintered diamond anvils. *American Mineralogist* 94 (2–3), 205–209. <http://dx.doi.org/10.2138/am.2009.2913>.
- Jacobs M.H.G., Oonk H.A.J., 2001. The Gibbs energy formulation of the α, β, and γ forms of Mg<sub>2</sub>SiO<sub>4</sub> using Grover, Getting and Kennedy's empirical relation between volume and bulk modulus. *Physics and Chemistry of Minerals* 28 (8), 572–585. <http://dx.doi.org/10.1007/s002690100180>.
- Jacobs M.H.G., Schmid-Fetzer R., 2010. Thermodynamic properties and equation of state of fcc aluminum and bcc iron, derived from a lattice vibrational method. *Physics and Chemistry of Minerals* 37 (10), 721–739. <http://dx.doi.org/10.1007/s00269-010-0371-6>.
- Jacobs M.H.G., Schmid-Fetzer R., Berg A.P., 2013. An alternative use of Kieffer's lattice dynamics model using vibrational density of states for constructing thermodynamic databases. *Physics and Chemistry of Minerals* 40 (3), 207–227. <http://dx.doi.org/10.1007/s00269-012-0562-4>.
- Jephcoat A.P., Hemley R.J., Mao H.K., 1988. X-ray diffraction of ruby (Al<sub>2</sub>O<sub>3</sub>:Cr<sup>3+</sup>) to 175 GPa. *Physica B+C* 150 (1–2), 115–121. [http://dx.doi.org/10.1016/0378-4363\(88\)90112-X](http://dx.doi.org/10.1016/0378-4363(88)90112-X).
- Kantor A., Kantor I., Merlini M., Glazyrin K., Prescher C., Hanfland M., Dubrovinsky L., 2012. High-pressure structural studies of eskolaite by means of single-crystal X-ray diffraction. *American Mineralogist* 97 (10), 1764–1770. <http://dx.doi.org/10.2138/am.2012.4103>.
- Kim-Zajonz J., Werner S., Shultz H., 1999. High pressure single crystal X-ray diffraction study on ruby up to 31 GPa. *Zeitschrift fur Kristallographie* 214 (6), 331–336. <http://dx.doi.org/10.1524/zkri.1999.214.6.331>.
- Kirby R.K., Hahn T.A., Rothrock B.D., 1972. Thermal expansion. In: B.H. Billings, D.E. Gray (Eds.), American Institute of Physics Handbook, 3rd edition. McGraw-Hill, New York, p. 4119–4142.
- Klemme S., O'Neill H.S.C., Schnelle W., Gmelin E., 2000. The heat capacity of MgCr<sub>2</sub>O<sub>4</sub>, FeCr<sub>2</sub>O<sub>4</sub>, and Cr<sub>2</sub>O<sub>3</sub> at low temperatures and derived thermodynamic properties. *American Mineralogist* 85 (11–12), 1686–1693. <http://dx.doi.org/10.2138/am-2000-11-1212>.
- Lin J.-F., Degtyareva O., Prewitt C.T., Dera P., Sata N., Gregoryanz E., Mao H.-K., Hemley R.J., 2004. Crystal structure of a high-pressure/high-temperature phase of alumina by in situ X-ray diffraction. *Nature Materials* 3 (6), 389–393. <http://dx.doi.org/10.1038/nmat1121>.
- Liu Y., Oganov A.R., Wang S., Zhu Q., Dong X., Kresse G., 2015. Prediction of new thermodynamically stable aluminum oxides. *Scientific Reports* 5, 9518. <http://dx.doi.org/10.1038/srep09518>.
- Logvinova A.M., Wirth R., Sobolev N.V., Seryotkin Y.V., Yefimova E.S., Floss C., Taylor L.A., 2008. Eskolaite associated with diamond from the Udachnaya kimberlite pipe, Yakutia, Russia. *American Mineralogist* 93 (4), 685–690. <http://dx.doi.org/10.2138/am.2008.2670>.
- Moore G.E., Kelly K.K., 1944. High-temperature heat contents of the chromium carbides and chromic oxide. In: US Bureau of Mines Technical Report, vol. 662, p. 10–15.
- Muan A., Sömiya S., 1959. Phase equilibrium studies in the system iron oxide-Al<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>. *Journal of the American Ceramic Society* 42 (12), 603–613. <http://dx.doi.org/10.1111/j.1151-2916.1959.tb13581.x>.
- Nakagiri N., Manghnani M.H., Ming L.C., Kimura S., 1986. Crystal structure of magnetite under pressure. *Physical and Chemistry of Minerals* 13 (4), 238–244. <http://dx.doi.org/10.1007/BF00308275>.
- Oganov A.R., Ono S., 2005. The high-pressure phase of alumina and implications for Earth's D'' layer. *Proceedings of the National Academy of Sciences* 102 (31), 10828–10831. <http://dx.doi.org/10.1073/pnas.0501800102>.
- Ono S., Ohishi Y., 2005. *In situ* X-ray observation of phase transformation in Fe<sub>2</sub>O<sub>3</sub> at high pressures and high temperatures. *Journal of Physics and Chemistry of Solids* 66 (10), 1714–1720. <http://dx.doi.org/10.1016/j.jpcs.2005.06.010>.
- Otero-de-la-Roza A., Luana V., 2011. Equations of state and thermodynamics of solids using empirical corrections in the quasiharmonic approximation. *Physical Review B* 84 (18), 184103. <http://dx.doi.org/10.1103/PhysRevB.84.184103>.
- Pushcharovsky Yu.M., Pushcharovsky D.Yu., 2010. Geology of the Earth's Mantle. GEOS Publishing House, Moscow, 140 p. (in Russian) [Пушаровский Ю.М., Пушаровский Д.Ю. Геология мантии Земли. М.: ГЕОС, 2010. 140 с.]
- Reichmann H.J., Jacobsen S.D., 2004. High-pressure elasticity of a natural magnetite crystal. *American Mineralogist* 89 (7), 1061–1066. <http://dx.doi.org/10.2138/am-2004-0718>.
- Richter P., Xu J.-A., Mao H.-K., 1988. Quasi-hydrostatic compression of ruby to 500 kbar. *Physical and Chemistry of Minerals* 16 (3), 207–211. <http://dx.doi.org/10.1007/BF00220687>.
- Ricolleau A., Fei Y., 2016. Equation of state of the high-pressure Fe<sub>3</sub>O<sub>4</sub> phase and a new structural transition at 70 GPa. *American Mineralogist* 101 (3), 719–725. <http://dx.doi.org/10.2138/am-2016-5409>.
- Robie R.A., Hemingway B.S., Fisher J.R., 1978. Thermodynamic Properties of Minerals and Related Substances at 298.15 K and 1 Bar Pressure and at Higher Temperatures. Washington, 456 p.

- Sato Y., Akimoto S., 1979. Hydrostatic compression of four corundum-type compounds:  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>, V<sub>2</sub>O<sub>3</sub>, Cr<sub>2</sub>O<sub>3</sub>, and  $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>. *Journal of Applied Physics* 50 (8), 5285. <http://dx.doi.org/10.1063/1.326625>.
- Saxena S.K., Shen G., 1992. Assessed data on heat capacity, thermal expansion and compressibility for some oxides and silicates. *Journal of Geophysical Research* 97 (B13), 19813–19825. <http://dx.doi.org/10.1029/92JB01555>.
- Schauer A., 1965. Thermal expansion, Gruneisen parameter, and temperature dependence of lattice vibration frequencies of aluminum oxide. *Canadian Journal of Physics* 43 (4), 523–531. <http://dx.doi.org/10.1139/p65-049>.
- Schultz A.H., Stubican V.S., 1970. Separation of phases by spinodal decomposition in the systems Al<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>, and Al<sub>2</sub>O<sub>3</sub>-Cr<sub>2</sub>O<sub>3</sub>-Fe<sub>2</sub>O<sub>3</sub>. *Journal of the American Ceramic Society* 53 (11), 613–616. <http://dx.doi.org/10.1111/j.1151-2916.1970.tb15984.x>.
- Shebanova O.N., Lazor P., 2003. Vibrational modeling of the thermodynamic properties of magnetite (Fe<sub>3</sub>O<sub>4</sub>) at high pressure from Raman spectroscopic study. *Journal of Chemical Physics* 119 (12), 6100–6110. <http://dx.doi.org/10.1063/1.1602072>.
- Shim S-H., Duffy T.S., Jeanloz R., Yoo C.-S., Iota V., 2004. Raman spectroscopy and x-ray diffraction of phase transitions in Cr<sub>2</sub>O<sub>3</sub> to 61 GPa. *Physical Review B* 69 (14), 144107. <http://dx.doi.org/10.1103/PhysRevB.69.144107>.
- Sivasubramanian K., Raju S., Mohandas E., 2001. Estimating enthalpy and bulk modulus from thermal expansion data – a case study with  $\alpha$ -Al<sub>2</sub>O<sub>3</sub> and SiC. *Journal of the European Ceramic Society* 21 (9), 1229–1235. [http://dx.doi.org/10.1016/S0955-2219\(00\)00323-X](http://dx.doi.org/10.1016/S0955-2219(00)00323-X).
- Skinner B.J., 1966. Section 6: Thermal expansion. In: S.P. Clark (Ed.), *Handbook of Physical Constants*. Geological Society of America Memoirs, vol. 97, p. 75–96. <http://dx.doi.org/10.1130/MEM97-p75>.
- Snow C.L., Lee C.R., Shi Q., Boerio-Goates J., Woodfield B.F., 2010. Size-dependence of the heat capacity and thermodynamic properties of hematite ( $\alpha$ -Fe<sub>2</sub>O<sub>3</sub>). *Journal of Chemical Thermodynamics* 42 (9), 1142–1151. <http://dx.doi.org/10.1016/j.jct.2010.04.009>.
- Sokolova T.S., Dorogokupets P.I., Dymshits A.M., Danilov B.S., Litasov K.D., 2016. Microsoft excel spreadsheets for calculation of P-V-T relations and thermodynamic properties from equations of state of MgO, diamond and nine metals as pressure markers in high-pressure and high-temperature experiments. *Computers and Geosciences* 94, 162–169. <http://dx.doi.org/10.1016/j.cageo.2016.06.002>.
- Sokolova T.S., Dorogokupets P.I., Litasov K.D., 2013. Self-consistent pressure scales based on the equations of state for ruby, diamond, MgO, B<sub>2</sub>-NaCl, as well as Au, Pt, and other metals to 4 Mbar and 3000 K. *Russian Geology and Geophysics* 54 (2), 181–199. <http://dx.doi.org/10.1016/j.rgg.2013.01.005>.
- Stixrude L., Lithgow-Bertelloni C., 2005. Thermodynamics of mantle minerals – I. Physical properties. *Geophysical Journal International* 162 (2), 610–632. <http://dx.doi.org/10.1111/j.1365-246X.2005.02642.x>.
- Strässle T., Klotz S., Kunc K., Pomjakushin V., White J.S., 2014. Equation of state of lead from high-pressure neutron diffraction up to 8.9 GPa and its implication for the NaCl pressure scale. *Physical Review B* 90 (1), 014101. <http://dx.doi.org/10.1103/PhysRevB.90.014101>.
- Tucek J., Machala L., Ono S., Namai A., Yoshikiyo M., Imoto K., Tokoro H., Ohkoshi S., Zboril R., 2015. Zeta-Fe<sub>2</sub>O<sub>3</sub> – A new stable polymorph in iron(III) oxide family. *Scientific Reports* 5, 15091. <http://dx.doi.org/10.1038/srep15091>.
- Umemoto K., Wentzcovitch R.M., 2008. Prediction of an U<sub>2</sub>S<sub>3</sub>-type polymorph of Al<sub>2</sub>O<sub>3</sub> at 3.7 Mbar. *Proceedings of the National Academy of Sciences* 105 (18), 6526–6530. <http://dx.doi.org/10.1073/pnas.0711925105>.
- Vinet P., Ferrante J., Rose J.H., Smith J.R., 1987. Compressibility of solids. *Journal of Geophysical Research* 92 (B9), 9319–9325. <http://dx.doi.org/10.1029/JB092iB09p09319>.
- Wachtman J.B., Scuderi T.G., Gleek G.W., 1962. Linear thermal expansion of aluminum oxide and thorium oxide from 100 to 1100 K. *Journal of the American Ceramic Society* 45 (7), 319–323. <http://dx.doi.org/10.1111/j.1151-2916.1962.tb11159.x>.
- Wessel C., Dronskowski R., 2013. A first-principles study on chromium sesquioxide, Cr<sub>2</sub>O<sub>3</sub>. *Journal of Solid State Chemistry* 199, 149–153. <http://dx.doi.org/10.1016/j.jssc.2012.12.019>
- Westrum E.F., Gronvold F., 1969. Magnetite (Fe<sub>3</sub>O<sub>4</sub>) heat capacity and thermodynamic properties from 5 to 350 K, low-temperature transition. *Journal of Chemical Thermodynamics* 1 (6), 543–557. [http://dx.doi.org/10.1016/0021-9614\(69\)90015-9](http://dx.doi.org/10.1016/0021-9614(69)90015-9).
- White G.K., Roberts R.B., 1983. Thermal expansion of reference materials: Tungsten and  $\alpha$ -Al<sub>2</sub>O<sub>3</sub>. *High Temperatures-High Pressures* 15 (3), 321–328.
- Wilburn D.R., Bassett W.A., Sato Y., Akimoto S., 1978. X ray diffraction compression studies of hematite under hydrostatic, isothermal conditions. *Journal of Geophysical Research* 83 (B7), 3509–3512. <http://dx.doi.org/10.1029/JB083iB07p03509>.
- Woodland A.B., Frost D.J., Trots D.M., Klimm K., Mezouar M., 2012. In situ observation of the breakdown of magnetite (Fe<sub>3</sub>O<sub>4</sub>) to Fe<sub>4</sub>O<sub>5</sub> and hematite at high pressures and temperatures. *American Mineralogist* 97 (10), 1808–1811. <http://dx.doi.org/10.2138/am.2012.4270>.
- Worlton T.G., Brugger R.M., Bennion R.B., 1968. Pressure dependence of the Néel temperature of Cr<sub>2</sub>O<sub>3</sub>. *Journal of Physics and Chemistry of Solids* 29 (3), 435–438. [http://dx.doi.org/10.1016/0022-3697\(68\)90120-0](http://dx.doi.org/10.1016/0022-3697(68)90120-0).

Zharkov V.N., Kalinin V.A., 1971. Equations of State of Solids at High Pressures and Temperatures. Consultants Bureau, New York, 257 p.

Ziemniak S.E., Anovitz L.M., Castelli R.A., Porter W.D., 2007. Thermodynamics of Cr<sub>2</sub>O<sub>3</sub>, FeCr<sub>2</sub>O<sub>4</sub>, ZnCr<sub>2</sub>O<sub>4</sub>, and CoCr<sub>2</sub>O<sub>4</sub>. *Journal of Chemistry Thermodynamics* 39 (11), 1474–1492. <http://dx.doi.org/10.1016/j.jct.2007.03.001>.



**Dorogokupets, Peter I.**, Doctor of Geology and Mineralogy, Head of Laboratory  
Institute of the Earth's Crust, Siberian Branch of RAS  
128 Lermontov street, Irkutsk 664033, Russia  
✉ e-mail: [dor@crust.irk.ru](mailto:dor@crust.irk.ru)

**Дорогокупец Петр Иванович**, докт. геол.-мин. наук, зав. лабораторией  
Институт земной коры СО РАН  
664033, Иркутск, ул. Лермонтова, 128, Россия  
✉ e-mail: [dor@crust.irk.ru](mailto:dor@crust.irk.ru)



**Sokolova, Tatiana S.**, Candidate of Geology and Mineralogy, Researcher  
Institute of the Earth's Crust, Siberian Branch of RAS  
128 Lermontov street, Irkutsk 664033, Russia  
Tel.: 8(3952)511680, e-mail: [sokolovats@crust.irk.ru](mailto:sokolovats@crust.irk.ru)

**Соколова Татьяна Сергеевна**, канд. геол.-мин. наук, н.с.  
Институт земной коры СО РАН  
664033, Иркутск, ул. Лермонтова, 128, Россия  
Тел.: 8(3952)511680, e-mail: [sokolovats@crust.irk.ru](mailto:sokolovats@crust.irk.ru)



**Dymshits, Anna M.**, Candidate of Geology and Mineralogy, Senior Researcher  
V.S. Sobolev Institute of Geology and Mineralogy, Siberian Branch of RAS  
3 Academician Koptyug ave., Novosibirsk 630090, Russia  
Tel.: 8(3833)303581, e-mail: [a.dymshits@gmail.com](mailto:a.dymshits@gmail.com)

**Дымшиц Анна Михайловна**, канд. геол.-мин. наук, с.н.с.  
Институт геологии и минералогии им. В.С. Соболева СО РАН  
630090, Новосибирск, пр. Академика Коptyуга, 3, Россия  
Тел.: 8(3833)303581, e-mail: [a.dymshits@gmail.com](mailto:a.dymshits@gmail.com)



**Litasov, Konstantin D.**, Doctor of Geology and Mineralogy, Chief Researcher, Professor of RAS  
V.S. Sobolev Institute of Geology and Mineralogy, Siberian Branch of RAS  
3 Academician Koptyug ave., Novosibirsk 630090, Russia  
Tel.: 8(3833)332517; e-mail: [klitasov@igm.nsc.ru](mailto:klitasov@igm.nsc.ru)

**Литасов Константин Дмитриевич**, докт. геол.-мин. наук, г.н.с., профессор РАН  
Институт геологии и минералогии им. В.С. Соболева СО РАН  
630090, Новосибирск, пр. Академика Коptyуга, 3, Россия  
Тел.: 8(3833)332517; e-mail: [klitasov@igm.nsc.ru](mailto:klitasov@igm.nsc.ru)