



## ON PROPER TIME OF THE SOURCE OF A STRONG EARTHQUAKE

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**ABSTRACT.** The physics of earthquakes was contributed to by the concept of proper time of the source of a strong earthquake, which is different from universal (calendar) time. The earlier idea of proper time was implicit and has been considered only in relation to the physics of aftershocks. The present paper extends the applicability of the concept of proper time, proposes a possible way of its measuring, and provides an example to illustrate the procedure for sequential ordering of earthquakes by proper time. The object of this study is a global activity of strong ( $M \geq 7$ ) earthquakes. We consider the sequence of earthquakes as a Poisson-type random process. Comparatively weak earthquakes are used as the "underground clock", the tick of which marks the proper time. The Poisson distribution is compared with the distributions for two sequences of strong earthquakes. One of the sequences is ordered by universal time, and another – by proper time. The studies indicate the distribution of events ordered by proper time is closer to the Poisson distribution than that of events ordered by universal time. We attribute this to the non-stationarity of the geological medium, which is an immanent property of the Earth's lithosphere.

**KEYWORDS:** geodynamics; aftershocks; Omori law; evolution equation; deactivation coefficient; non-uniformity of time; non-stationarity of geological medium; Poisson process

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## SHORT COMMUNICATION

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## 1. INTRODUCTION

In a broad sense, the proper time implies natural mechanisms of synchronization of the dynamic system plunged into the non-stationary medium. There are known cases of a proper choice of "internal clock" actually regulating certain aspects of the functioning of dynamic system. For example, in biology, life cycles of organisms are sometimes regulated by circadian rhythm whose origin is attributed to the Earth's rotation. In solar-terrestrial physics, the Sun's rotation leads to the fact that the Earth quasiperiodically crosses sharp boundaries between the sectors of the interplanetary magnetic field which promotes the rational ordering of geoelectromagnetic events in their time sequence. In this paper we made an attempt to introduce an idea of proper time into tectonophysics. A few examples should clarify the situation.

The Earth's crust is characterized by continuous occurrence of complex tectonic processes causing deformation and change in the stress state of the medium which periodically exhibits structural reconstruction, major fault occurrence, and main earthquake shock [Gzovsky, 1975; Goncharov et al., 2005]. In other words, an earthquake occurs in the non-stationary medium whose parameters vary over time. From physics we know that the mode of functioning of dynamic system as a whole and disruptive transition in particular in the non-stationary medium depend largely on the rate of change in parameters of the medium in which the system is plunged. It should be emphasized that the mathematical, laboratory and natural methods and results of tectonophysical modeling known so far allow providing qualitative representation of the non-stationarity of these rocks, but this is not enough for optimal chronologization of strong earthquake flow. In these circumstances, the idea of proper time which is counted from weaker earthquake shocks can in some cases be useful.

The idea of the proper time of the earthquake source arose during the study of aftershocks of a strong earthquake [Guglielmi, 2016, 2017]. The idea turned out to be successful and was used in a number of publications [Zavyalov et al., 2020; Guglielmi et al., 2021, 2022; Guglielmi, Zotov, 2021; Zotov, Guglielmi, 2021]. The history of the issue is as follows.

In 1894, twenty-six-year-old Fusakichi Omori discovered the first law of the physics of earthquakes [Omori, 1895], which says that the aftershock frequency  $n$ , on average, hyperbolically decreases with time:

$$n(t) = \frac{k}{c+t}, \quad (1)$$

where  $k>0$ ,  $c>0$ ,  $t>0$ . It was noticed [Guglielmi, 2016], that the Omori law (1) can be presented in the form of equation describing the aftershock evolution:

$$\frac{dn}{dt} + \sigma n^2 = 0, \quad (2)$$

where  $\sigma$  is the so called deactivation coefficient of an earthquake source "cooling down" after the main shock. At  $\sigma=\text{const}$ ,

the general solution of differential equation (2) coincides up to a sign with Omori formula (1).

The Omori law was viewed in different ways by P. Hirano [Hirano, 1924], T. Utsu, and others [Utsu 1961; Utsu et al., 1995]. They thought formula (1) to be incorrect and proposed their own version of the law:

$$n(t) = \frac{k}{(c+t)^p}, \quad (3)$$

and we give preference to law version (2). It is formulated only by using three symbols ( $n$ ,  $t$ ,  $\sigma$ ) instead of five ( $n$ ,  $t$ ,  $k$ ,  $c$ ,  $p$ ) used in formula (3). Moreover, evolution equation (2) allows for natural generalizations [Zavyalov et al., 2020; Guglielmi et al., 2022; Guglielmi, Klain, 2020], which are not at all evident in Hirano-Utsu formulation of the Omori law.

We write the solution of equation (2) in the form closest to Omori formula (1):

$$n(\tau) = \frac{1}{n_0^{-1} + \tau}, \quad (4)$$

where  $n_0=n(0)$ . The value of

$$\tau = \int_0^t \sigma(t') dt' \quad (5)$$

will be used as **the proper time** of the earthquake source "cooling down" after the main shock. Here, we follow the tradition of denominating the proper time by  $\tau$ , in order to distinguish it from universal time  $t$ . The experimentally observed difference between  $\tau$  and  $t$  can be attributed to the non-stationarity of a rock massif in the source after the major fault occurrence.

To date, a successful experience in the use of  $\tau$  along with universal time  $t$  has been accumulated during the study of aftershocks [Zavyalov et al., 2020; Guglielmi et al., 2019, 2021, 2022; Guglielmi, Zotov, 2021; Zotov, Guglielmi, 2021]. Emphasis should be on the fact that previously the idea of proper time was implicit and rather narrow in scope related to the physics of aftershocks. In this paper, we will try to enlarge on the idea of proper time of occurrence of tectonic processes in the hope that it will be useful in the future. Our attempt is justified by the fact that the non-stationarity is an immanent property of the Earth's lithosphere.

## 2. GENERALIZATION

The object of the research is the global activity of strong  $M \geq 7$  earthquakes. In theory, there should be the "underground clock", the tick of which marks the course of proper time. The form of formula (4) suggests that it is reasonable to try to use relatively weak earthquakes for counting proper time. The  $6 \leq M < 7$  earthquakes will be chosen as a test run.

Here is a small experiment to illustrate how the proper time can be used. The earthquake sequence will be represented as the Poisson random process, i.e. as a chain of instantaneous events separated by certain intervals of time. The Poisson distribution has the form:

$$p_k = \frac{\lambda^k}{k!} \exp(-\lambda). \quad (6)$$

At  $\lambda = at$ , the  $p_k(t)$  value expresses the probability that the number of events  $k$  occurs within the time interval  $t$ . The average number of events is

$$\langle k \rangle = \sum_{k=0}^{\infty} k p_k(t) = at. \quad (7)$$

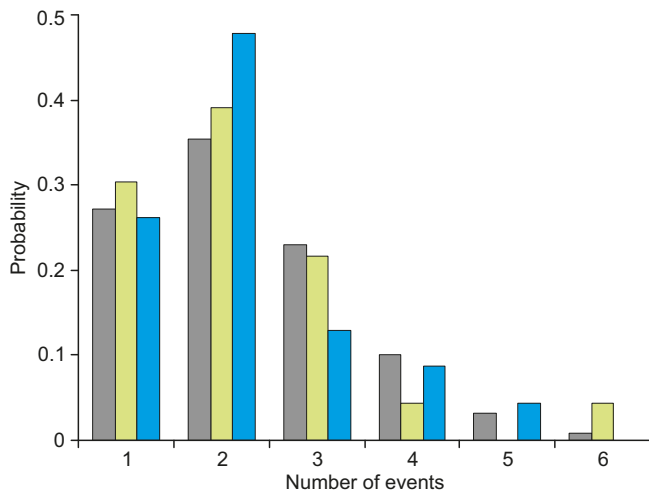
At  $\lambda = \alpha\tau$ , the  $p_k(\tau)$  value expresses the probability that the number of events  $k$  occurs within the time interval  $\tau$ . The average number of events is

$$\langle k \rangle = \sum_{k=0}^{\infty} k p_k(\tau) = \alpha\tau. \quad (8)$$

We compare distribution (6) with the distributions for two strong earthquake sequences. One of the sequences

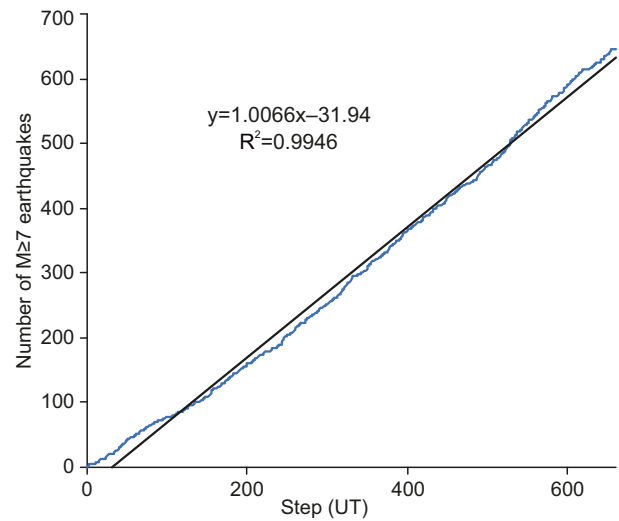
will be ordered by universal time, and another – by proper time. The test result is shown in Fig. 1. The data on the temporal distribution of earthquakes in 2020 and 2021 were taken from the USGS/NEIC Earthquake Catalog (<https://earthquake.usgs.gov>). Over 24 months, there occurred 31 strong earthquakes, i.e.  $\lambda = 1.3$  events in an average month. Over 24 months, there were observed 255 weaker earthquakes which are to be used for counting proper time.

The Poisson distribution in Fig. 1 is shown in gray. Blue and green colors show the distributions of events ordered by universal time and proper time, respectively. Generally, the distribution of events ordered by proper time is a little closer to the Poisson distribution than that of the events ordered by universal time. However, the sampling size in this illustrative example is too small to be given special importance at this stage of the study. In the next section, we

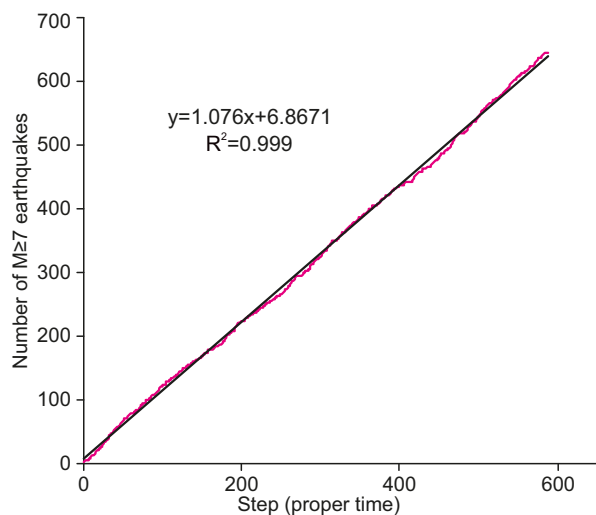


**Fig. 1.** Distributions of strong earthquakes.

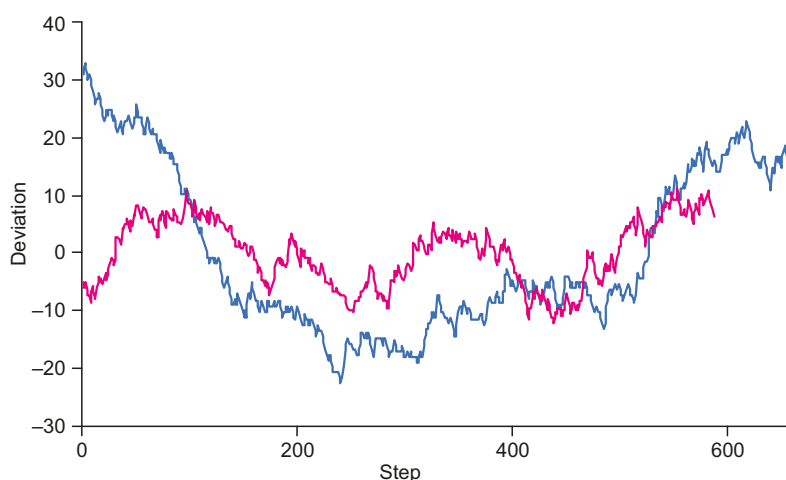
The horizontal axis shows the number of events in one-month interval according to universal time (blue color) and in the interval of proper time (green color). The gray color represents the Poisson distribution.



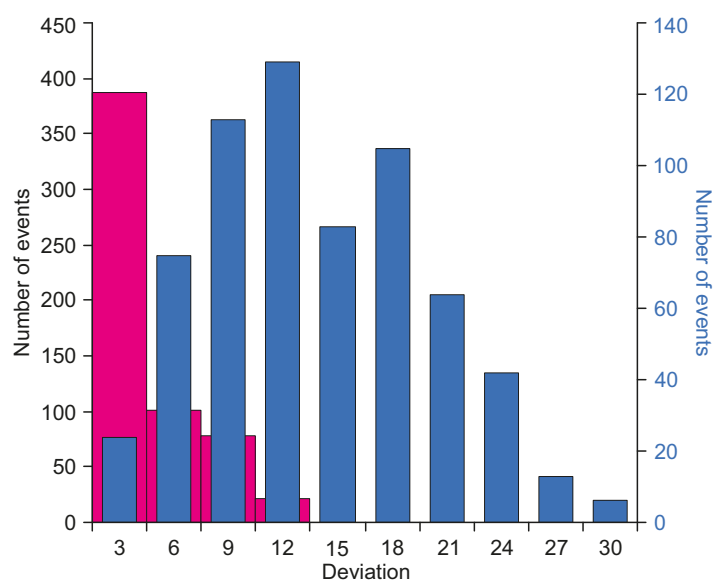
**Fig. 2.** Dependence of the accumulated number of  $M \geq 7$  earthquakes on universal time. The time step of 26 days is chosen so that the number of points approximately coincides with the number of points in Fig. 3.



**Fig. 3.** Dependence of the accumulated number of  $M \geq 7$  earthquakes on proper time. The accumulation step is equal to 10 strokes of the "underground clock".



**Fig. 4.** Deviations of the curves, shown in Fig. 2 (blue) and Fig. 3 (red), from the theoretical straight line.



**Fig. 5.** Distributions of the absolute values of the deviations of the curves, shown in Fig. 2 (blue) and Fig. 3 (red), from the straight lines.

propose the evolving plan for a more thorough study of the sequence of events ordered by proper time.

We make a comparative analysis based on more numerical data. First, note that in the Poisson process the events on average occur on a regular basis. This implies that over time the Poisson process leads, on average, to a linear growth in the accumulated number of events.

From 1973 to 2019, there occurred 646 earthquakes with magnitudes  $M \geq 7$  and 5886 earthquakes with magnitudes  $6 \leq M < 7$ . The process of accumulation of strong earthquakes is shown in Fig. 2 and Fig. 3, with the events ordered by universal time and proper time, respectively. We see that in the second case the experimental curve exhibits lesser deviation from the straight line than in the first case.

Fig. 4 emphasizes this difference showing deviations of real curves from the straight line. It is obvious that the use of proper time fits the Poisson theory better than the use of universal time.

It is even more convincingly shown in Fig. 5, which exhibits the distributions of absolute deviations of experimental curves from theoretical lines.

### 3. DISCUSSION

The geological medium is generally non-stationary; its parameters vary with time. Rather important is the fact the non-stationarity is often hidden from view. Studying geodynamic phenomena by the universal time clock, the observer reveals some correlations, which, however, in non-stationary latent conditions, may be weak or remain completely unobserved when the phenomenon sequence is ordered by universal time. In these circumstances, a less perfect clock, counting the proper time, can be a useful research tool.

The hyperbolic Omori law in form (2) allows us to provide an illustrative example explaining of the above mentioned. The proper time is counted on aftershocks. The we introduce auxiliary function  $g(t)=1/n(t)$  and average

it over properly chosen small intervals of time:  $g \rightarrow \langle g \rangle$ . After that, the source deactivation factor is calculated by the formula:

$$\sigma = \frac{d}{dt} \langle g \rangle. \quad (9)$$

In the so called Omori epoch [Zavyalov et al., 2020], when  $\sigma = \text{const}$ , there is observed a uniform increase of the parameter  $\langle g \rangle$  with time:  $\langle g \rangle \propto t$ .

We described briefly the process of formulating and solving inverse problems in earthquake source physics [Guglielmi, 2017; Zavyalov et al., 2020; Guglielmi et al., 2019, 2021]. The experience showed that the deactivation factor – an important phenomenological earthquake source parameter, – undergoes complex temporal variations. In other words, rocks of the earthquake source are in the non-stationary state. Note that in the process of formulating and solving inverse problems the idea of proper time was implicitly used.

P. Hirano [Hirano, 1924] had previously paid attention to the non-stationary geological medium in "cooling" source and tried to take into account the non-stationarity, having substituted the parameter  $p$  in formula (3) by sectionally continuous function  $p(t)$ . However, such substitution makes formula (3) logically obscure. In contrast to this, the Omori law in form (2), (4) is mathematically correct, embodies the concept of proper time, and allows flexible modeling of the non-stationary medium in the earthquake source.

When trying to generalize the concept of proper time, we chose the "clock", which, so to say, ticks too loud. In fact, the time counting by recording  $6 \leq M < 7$  earthquakes is only motivated by the simplicity of processing of a relatively small numerical dataset. In the future we plan to use weaker earthquakes.

A choice of specific research object (a global strong earthquake activity) was made rather arbitrarily. In future research it would be interesting to study not only global but also regional seismicity. The subject of research might involve, for example, the cross-correlation of earthquakes in Northern and Southern California. From 1983 to 2007, it was discovered that there was an anticorrelation of fluctuations in the average daily magnitude of earthquakes in these two adjacent regions [Zotov et al., 2022]. This result can be made more explicit when the earthquakes are ordered by proper time.

#### 4. CONCLUSION

This paper of purely debatable nature explicitly contributes to the geotectonic notion of proper time, different from universal time. The idea of proper time we and our colleagues had previously dealt with was implicit and relatively narrow in scope related to the physics of aftershocks. In this paper we tried to extend the applicability of the concept of proper time, showed a probable way of its measuring, and gave a simple example of the process in which strong earthquake sequences are ordered by proper time.

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#### 6. CONTRIBUTION OF THE AUTHORS

All authors made an equivalent contribution to this article, read and approved the final manuscript.

#### 7. DISCLOSURE

The authors declare that they have no conflicts of interest relevant to this manuscript.

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